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MORE ON ALMOST STRONGLY-θ-β-CONTINUOUS FUNCTIONS AND CONTRA ALMOST-β-CONTINUOUS FUNCTIONS

BY

SANJAY TAHILIANI

Abstract. β-continuity was introduced by Abd El-Monsef et al. and then it's weak forms are also studied. In this paper we continued our study on almost strongly-θ-β-continuous functions introduced by Tahiliani and contra almost-β-continuous introduced by Baker.

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1. Introduction

The concept of β-open sets and β-continuity was introduced and studied by Abd El-Monsef et al. [1]. Further Popa and Noiri [20] introduced weak β-continuity and subsequently Noiri and Popa [14] introduced the concept of almost β-continuity. Most recently, Baker [4] introduced the concept of contra almost-β-continuous functions which is stronger than weak β-continuous functions. In this paper we continue our study on almost strongly-θ-β-continuous functions introduced by Tahiliani [26] and which is stronger than almost β-continuous functions. It is also seen that contra almost β-continuous functions are independent of almost β-continuous functions.

2. Preliminaries

Throughout the present paper, X and Y denote topological spaces. Let

A be a subset of X. We denote the interior and closure of A by $\mathrm{Int}(A)$ and $\mathrm{Cl}(A)$ respectively. A subset A of a space X is said to be regular open if $A = \mathrm{Int}(\mathrm{Cl}(A))$ and it's complement is said to be regular closed if $A = \mathrm{Cl}(\mathrm{Int}(A))$. A subset A of a space X is called semi-open [10] (resp. β-open [1]) if $A \subseteq \mathrm{Cl}(\mathrm{Int}(A))$ (resp. $A \subseteq \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(A)))$). The family of all β-open sets containing x will be denoted by $\beta O(X, x)$. The complement of β-open set is β-closed [1]. The intersection of all β-closed sets containing A is called β-closure of A and is denoted by $\beta\mathrm{Cl}(A)$. A point x of X is called a semipre-θ-cluster point [16] or β-θ-cluster point of A if $\beta\mathrm{Cl}(U) \cap A \neq \emptyset$ for every β-open set U of X containing x. The set of all β-θ-cluster point of A is said to be semipre-θ-closure [16] or β-θ-closure of A and is denoted by $\beta\mathrm{Cl}_\theta(A)$. A subset A is said to be β-θ-closed if $A = \beta\mathrm{Cl}_\theta(A)$. The complement of a β-θ-closed set is said to be β-θ-open. A point x is in the β-θ-interior of A denoted by $\beta\ \mathrm{Int}_\theta(A)$, if there exists β-open set U such that $\beta\mathrm{Cl}(U) \subseteq A$. A subset A of a space X is said to be β-clopen [16] if it is β-open and β-closed.

Definition 2.1. A function $f : X \to Y$ is called *β-continuous* [1] (resp. *almost β-continuous* [14], *weakly β-continuous* [20]) if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists a β-open set U such that $f(U) \subseteq V$(resp. $f(U) \subseteq \mathrm{Int}(\mathrm{Cl}(V)), f(U) \subseteq \mathrm{Cl}(V)$).

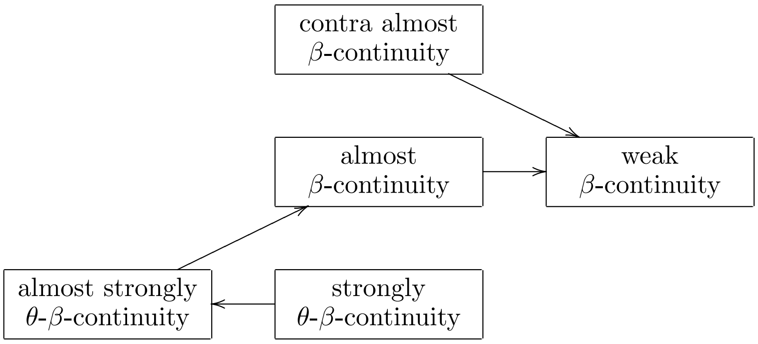
Definition 2.2. A function $f : X \to Y$ is said to be *strongly θ-β-continuous* [17] (resp. *almost strongly θ-β-continuous* [26]) if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists $U \in \beta O(X, x)$ such that $f(\beta\mathrm{Cl}(U)) \subseteq V$ (resp. $f(\beta\mathrm{Cl}(U)) \subseteq \beta\ \mathrm{Cl}(V)$).

Definition 2.3. A function $f : X \to Y$ is said to be *contra almost β-continuous* [4] if the inverse image of each regular open set in Y is β-closed in X.

Every contra almost β-continuous function is weakly β-continuous ([4], Theorem 3.3). Based on the above definitions, we have the following diagram (see next page). We note that none of the reverse implications of that diagram are true, as can be seen in following examples.

Example 2.1. Every weakly β-continuous function may not be contra almost β-continuous ([4], Example 3.2). Also almost strongly-θ-β-continuous function may not be strongly-θ-β-continuous ([26], Example

2.5). Moreover weak β -continuity does not imply almost β-continuity ([20], page 84).



Example 2.2. Let $X = \{a, b, c\}$ and let $\tau = \{\emptyset, X, \{b, c\}, \{a, c\}, \{c\}\}$ and $\sigma = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ be the topologies on X. Let $f : (X, \tau) \to (X, \sigma)$ be the identity function. Then f is contra almost β-continuous but not almost β-continuous, not even almost strongly-θ-β-continuous.

Remark 2.1. Contra almost β-continuity and almost β-continuity are independent concepts as can be seen from Example 2.2 and the following example:

Example 2.3. Let $X = \{a, b, c\}$ and let $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$ and $\sigma = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$. Let $f : (X, \tau) \to (X, \sigma)$ be the identity function. Then f is almost β-continuous but not contra almost β-continuous.

3. Almost-strongly-θ-β-continuous functions

Lemma 3.1. *$\beta Cl(A)$ is β-clopen for every β-open subset A of X.*

Proof. It is obvious from the fact that $\beta\mathrm{Cl}(A)$ is β-closed and $\beta\mathrm{Cl}(A)$ is β-open if A is β-open. ([8], Lemma 4.1).

The family of regular open sets of a space (X, τ) forms a base for a smaller topology τ_s on X, called *semi-regularization* of τ. □

Theorem 3.1. *The following are equivalent for a function $f : X \to Y$:*
(1) *f is almost strongly θ-β-continuous,*

(2) *for each* $x \in X$ *and each open set* V *of* Y *containing* $f(x)$*, there exists a* β*-clopen set* U *containing* x *such that* $f(U) \subseteq \mathrm{Int}(\mathrm{Cl}(V))$*,*

(3) $f^{-1}(V) \subseteq \beta\ \mathrm{Int}_\theta(f^{-1}\ (\mathrm{Int}(\mathrm{Cl}(V)))$ *for every open set* V *of* Y*,*

(4) $f : X \to Y_s$ *is strongly-*θ*-*β*-continuous, where* Y_s *denotes the semi-regularization of* Y*.*

Proof. (1)$\Rightarrow$(2). It is obvious by ([26], Theorem 2.1.2) and Lemma 3.1.

(2)$\Rightarrow$(3). Suppose that V is an open set in Y and that $x \in f^{-1}(V)$. Then $f(x) \in V$. By (2), there exists a β-clopen set U such that $f(U) \subseteq \mathrm{Int}(\mathrm{Cl}(V))$. Therefore, we have $x \in U = \beta\mathrm{Cl}(U) \subseteq f^{-1}\ (\mathrm{Int}(\mathrm{Cl}(V)))$ and hence $x \in \beta\ \mathrm{Int}_\theta(f^{-1}\ (\mathrm{Int}(\mathrm{Cl}(V))))$. It follows that $f^{-1}(V) \subseteq \beta\ \mathrm{Int}_\theta(f^{-1}\ (\mathrm{Int}(\mathrm{Cl}(V))))$.

(3)$\Rightarrow$(4). Let $x \in X$ and let V be any open set of Y_s containing $f(x)$. There exists a regular open set G of Y such that $f(x) \in G \subseteq V$. By (3), we have $x \in f^{-1}(G) \subseteq \beta\ \mathrm{Int}_\theta(f^{-1}(G))$ and hence there exists $U \in \beta O(X)$ such that $x \in U \subseteq \beta\mathrm{Cl}(U) \subseteq f^{-1}(G)$. Therefore, we obtain $f(\beta\mathrm{Cl}(U)) \subseteq V$. This shows that $f : X \to Y_s$ is strongly-θ-β-continuous.

(4)$\Rightarrow$(1). Let V be any regular open set of Y. For any $x \in f^{-1}(V)$, $f(x) \in V$ and V is open in Y_s. There exists $U_x \in \beta O(X)$ such that $f(\beta\mathrm{Cl}(U_x)) \subseteq V$ and hence $\beta\mathrm{Cl}(U_x) \subseteq f^{-1}(V)$. Therefore,we have $f^{-1}(V) \subseteq \beta\mathrm{Int}_\theta(f^{-1}(V))$ and $f^{-1}(V)$ is β-θ-open in X. It follows from ([26], Corollary 2.1.2) that f is almost strongly θ-β-continuous. $\square$

Definition 3.1. A space X is said to be

(i) *weakly-*T_2 [24] if each point of X is expressed by the intersection of regular closed sets of X,

(ii) rT_0 [9] if for any pair of distinct points in X, there exists a regular open set containing one point but not the other,

(iii) β-T_2 [11] if for each pair of distinct points x and y in X, there exists disjoint β-open sets U and V containing x and y respectively.

Remark 3.1. From Definition 3.1, we have the following diagram:

$$\text{Hausdorff} \longrightarrow \text{weakly-}T_2 \longrightarrow rT_0 \longrightarrow \beta\text{-}T_2$$

Theorem 3.2. *If* $f : X \to Y$ *is an almost strongly* θ*-*β*-continuous injection and* Y *is* rT_0*, then* X *is* β*-*T_2*.*

Proof. Let x and y be distinct points of X. By the injectivity of f, it follows that $f(x) \neq f(y)$ and there exists either a regular open set V

containing $f(x)$ not containing $f(y)$ or a regular open set W containing $f(y)$ not containing $f(x)$. If the first case holds, then there exists $U \in \beta O(X)$ such that $f(\beta \mathrm{Cl}(U)) \subseteq V$. Therefore, we obtain $f(y) \notin f(\beta \mathrm{Cl}(U))$ and hence $X \setminus \beta \mathrm{Cl}(U)$ is β-clopen in X containing y by Lemma 3.1. If the second case holds, then we obtain the similar result. Therefore X is β-T_2 ([18], Theorem 3.1). □

Corollary 3.1. *If $f : X \to Y$ is almost strongly θ-β-continuous injection and Y is weakly-T_2, then X is β-T_2.*

Corollary 3.2 ([26], Theorem 4.7)**.** *If $f : X \to Y$ is almost strongly θ-β-continuous injection and Y is Hausdorff, then X is β-T_2.*

Recall that a space X is *submaximal* if every dense set is open and it is said to be *extremally disconnected* if the closure of every open set is open.

Lemma 3.2 ([8])**.** *If X is submaximal and extremally disconnected, then every β-open set in X is open.*

Remark 3.2. In the view of above Lemma, we can conclude that If X is submaximal and extremally disconnected, then every β-clopen set is clopen.

Let (X, τ) be a topological space. The quasi-topology on X is the topology having base as clopen subsets of (X, τ). The open (resp. closed) subsets of the quasi-topology are said to be *quasi-open* (resp. *quasi-closed*).

Theorem 3.3. *If $f, g : X \to Y$ are almost strongly θ-β-continuous functions, X is submaximal and extremally disconnected and Y is Hausdorff, then $E = \{x | f(x) = g(x)\}$ is quasi-closed in X.*

Proof. If $x \in X \setminus E$,then it follows that $f(x) \neq g(x)$. Since Y is Hausdroff, there exists open sets V and W containing $f(x)$ and $g(x)$, respectively such that $\mathrm{Int}(\mathrm{Cl}(V)) \cap \mathrm{Int}(\mathrm{Cl}(W)) = \emptyset$. Since f and g are almost strongly θ-β-continuous, there exists $U \in \beta O(X, x)$ and $G \in \beta O(X, x)$ such that $f(\beta \mathrm{Cl}(U)) \subseteq \mathrm{Int}(\mathrm{Cl}(V))$ and $g(\beta \mathrm{Cl}(G)) \subseteq \mathrm{Int}(\mathrm{Cl}(W))$. Let $D = \beta \mathrm{Cl}(U) \cap \beta \mathrm{Cl}(G)$. Then by Remark 3.2, D is clopen in X as X is submaximal and extremally disconnected. Since $f(D) \cap g(D) = \emptyset, D \cap E = \emptyset$ and $X \setminus E$ is quasi-open. Therefore E is quasi-closed. □

Definition 3.2. A function $f : X \to Y$ is said to be

(i) *δ-continuous* [13] if for each $x \in X$ and each regular open set V of Y containing $f(x)$, there exists a regular open set U of X containing x such that $f(U) \subseteq V$,

(ii) *β-clopen* if the image of each β-clopen subset U of X, $f(U)$ is β-clopen in Y.

Theorem 3.4. *If $f : X \to Y$ is an almost strongly θ-β-continuous function and $g : Y \to Z$ is δ-continuous, then $g \circ f : X \to Z$ is almost strongly θ- β-continuous.*

Proof. Let $x \in X$ and W be a regular open set in Z containing $(g \circ f)(x)$. Since g is δ-continuous, there exists a regular open set V of Y containing $f(x)$ such that $g(V) \subseteq W$. Since f is almost strongly θ-β-continuous, there exists a β-open set U such that $f(\beta\mathrm{Cl}(U)) \subseteq V$. This shows that $(g \circ f)(\beta\mathrm{Cl}(U)) \subseteq W$. Therefore, $g \circ f$ is almost strongly θ-β-continuous.□

Theorem 3.5. *Let $f : X \to Y$ and $g : Y \to Z$ be functions. If $g \circ f : X \to Z$ is an almost strongly θ-β-continuous function and f is a β-clopen surjection, then g is almost strongly θ-β-continuous.*

Proof. Suppose that $y \in Y$ and $f(x) = y$. Let V be an open set of Z containing $g(f(x))$. Since $g \circ f$ is almost strongly θ-β-continuous, there exists β-open set U such that $g(f(\beta\mathrm{Cl}(U))) = (g \circ f)(\beta\mathrm{Cl}(U)) \subseteq \beta\mathrm{Cl}(V) = \mathrm{Int}(\mathrm{Cl}(V))$. Since f is β-clopen and $\beta\mathrm{Cl}(U)$ is β-clopen, $f(\beta\mathrm{Cl}(U))$ is β-clopen in Y containing y. By Theorem 3.1, g is almost strongly θ-β-continuous. □

Theorem 3.6. *If a function $f_i : X \to Y_i$ is almost strongly θ-β-continuous for each $i \in I$, then the product function $f : \prod X_i \to \prod Y_i$, defined by $f(x) = (f_i(x_i))$ for each $x = (x_i) \in \prod X_i$ is almost strongly θ-β-continuous.*

Proof. Let $x \in \prod X_i$ and W be any open set of $\prod Y_i$ containing $f(x)$. Then there exists a open set V_{ij} of Y_i such that $f(x) = (f_i(x_i)) \in \prod_{i=1}^{n} V_{ij} \times \prod_{i \neq ij} Y_i \subseteq W$. Since f_i is almost strongly θ-β-continuous function, by Theorem 3.1, there exists β-clopen set U_{ij} such that $f_{ij}(U_{ij}) \subseteq \beta\mathrm{Cl}(V_{ij})$ for $j = 1, 2, \ldots, n$. Put $U = \prod_{j=1}^{n} U_{ij} \times \prod_{i \neq i_j} X_i$. Then it follows from ([3], Lemma 2.2) that U is β-open in $\prod X_i$. By a similar argument, we obtain that U is β-clopen in $\prod X_i$ containing x. Moreover by Lemma 3.1, we have

$f(U) \subseteq \prod_{j=1}^{n} fi_j(Ui_j) \times \prod_{i \neq i_j} Y_i \subseteq \prod_{j=1}^{n} \beta \ \mathrm{Cl}(Vi_j) \times \prod_{i \neq i_j} Y_i \subseteq \beta\mathrm{Cl}(W)$. It follows from Theorem 3.1 that f is almost strongly θ-β-continuous. □

Theorem 3.7. *If a function $f : X \to \prod Y_i$ is almost strongly θ-β-continuous and $p_i : \prod Y_i \to Y_i$ is the natural projection, then $p_i \circ f : X \to Y_i$ is almost strongly θ-β-continuous.*

Proof. This is an immediate consequence of Theorem 3.4 since every open function is δ-continuous ([13], Theorem 4.11). □

4. Some properties of contra almost β-continuous functions

Theorem 4.1. *Let (X, τ) and (Y, σ) be topological spaces. The following statements are equivalent:*

(1) *f is contra almost β-continuous,*

(2) *$f^{-1}(V)$ is β-open in X for every regular closed set V in Y,*

(3) *for each $x \in X$ and each regular closed set V in Y containing $f(x)$, there exists a β-open set U in X containing x such that $f(U) \subseteq V$,*

(4) *for each $x \in X$ and each regular open set V in Y non containing $f(x)$, there exists a β-closed set K in X non containing x such that $f^{-1}(V) \subseteq K$,*

(5) *f^{-1} $(\mathrm{Int}(\mathrm{Cl}(G)))$ is β-closed in X for every open subset G of Y,*

(6) *f^{-1} $(\mathrm{Cl}(\mathrm{Int}(F)))$ is β-open in X for every closed subset F of Y.*

Proof. (1)⇔(2), (2)⇒(3) Obvious.

(3)⇒(2) Let V be any regular closed set in Y and $x \in f^{-1}(V)$. From (3), there exists a β-open set U_x in X containing x such that $U \subseteq f^{-1}(V)$. We have $f^{-1}(V) = \bigcup\{U_x : x \in f^{-1}(V)\}$. Thus $f^{-1}(V)$ is β-open.

(3)⇔(4) Let V be any regular open set in Y not containing $f(x)$. Then, $Y \setminus V$ is a regular closed set containing $f(x)$. By (3), there exists a β-open set U in X containing x such that $f(U) \subseteq Y \setminus U$. Hence, $U \subseteq f^{-1}(Y \setminus V) \subseteq X \setminus f^{-1}(V)$ and then $f^{-1}(V) \subseteq X \setminus U$. Take $K = X \setminus U$. Then K is a β-closed set in X not containing x. Converse can be shown easily.

(1)⇔(5) Let G be open subset of Y. Since $\mathrm{Int}(\mathrm{Cl}(G))$ is regular open, then by (1), it follows that f^{-1} $(\mathrm{Int}(\mathrm{Cl}(G)))$ is β-closed in X. Converse can be shown easily.

(2)⇔(6) It can be obtained similarly as in (1)⇔(5). □

Theorem 4.2. *Let Y be extremally disconnected. Then the function $f : X \to Y$ is contra almost β-continuous if and only if it is almost β-continuous.*

Proof. (Necessity). Let $x \in X$ and let V be any regular open set of Y containing $f(x)$. Since Y is extremally disconnected, by ([19], Lemma 5.6), V is clopen and hence V is regular closed. By Theorem 4.1, there exists $U \in \beta O(X, x)$ such that $f(U) \subseteq V$. By ([15], Corollary 2), f is almost β-continuous.

(Sufficiency). Let F be any regular closed set of Y. Since Y is extremally disconnected, F is regular open and $f^{-1}(F)$ is β-open in X. This shows that f is contra almost β-continuous.

A topological space X is said to be *almost regular* [22] if for each regular closed set A of X and each $x \in X \setminus A$, there exists disjoint open sets U and V of X such that $x \in U$ and $A \subset V$. □

Theorem 4.3. *If a function $f : X \to Y$ is contra almost β-continuous and Y is almost regular, then f is almost β-continuous.*

Proof. It's clear from ([14], Theorem 6). □

Definition 4.1. A function $f : X \to Y$ is said to be *contra* R-*continuous* [7] if the inverse image of each regular open set in Y is regular closed in X.

Theorem 4.4. *Let $f : X \to Y$ and $g : Y \to Z$ be two functions. If f is contra almost β-continuous and g is contra R-continuous, then $g \circ f$ is almost β-continuous.*

Proof. Let V be any regular open in Z. Since g is contra R-continuous, $g^{-1}(V)$ is regular closed in Y. Now since f is contra almost β-continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is β-open in X. Hence $g \circ f$ is almost β-continuous. □

Definition 4.2. A filter base $\wedge$ is said to be *β-convergent* (resp. *rc-convergent*) to a point x in X if for any $U \in \beta O(X, x)$ (resp. regular closed set U) containing x), there exists a $B \in \wedge$ such that $B \subseteq U$.

The following are the obvious Theorems

Theorem 4.5. *If a function $f : X \to Y$ is contra almost β-continuous, then for each point $x \in X$ and each filter base $\wedge$ in $X\beta$-converging to x, the filter base $f(\wedge)$ is rc-convergent to $f(x)$.*

Theorem 4.6. *Let $f : X \to Y$ be a function and $x \in X$. If there exists $U \in \beta O(X)$ such that $x \in U$ and the restriction of f to U is contra almost β-continuous at x, then f is contra almost β-continuous at x.*

Now, we investigate the relationships among almost contra-β -continuous functions, connectedness and compactness.

Definition 4.3. A topological space X is called *β-ultra-connected* if every two non empty β-closed subsets of X intersect.

Definition 4.4. A topological space X is called *hyperconnected* [25] if every open set is dense.

Theorem 4.7. *If X is β-ultra-connected and $f : X \to Y$ is contra almost β-continuous and surjective, then Y is hyperconnected.*

Proof. Assume that Y is not hyperconnected. So there exists an open set V such that V is not dense in Y. Hence there exists disjoint non empty regular open subsets B_1 and B_2 in Y, namely $\mathrm{Int}(\mathrm{Cl}(V))$ and $Y\setminus \mathrm{Cl}(V)$. Since f is almost contra β-continuous and onto, $A_1 = f^{-1}(B_1)$ and $A_2 = f^{-1}(B_2)$ are disjoint non empty β-closed subsets of X. By assumption, the β-ultra-connectedness of X implies that A_1 and A_2 must intersect. By contradiction, Y is hyperconnected. □

Definition 4.5. A space X is said to be

(i) *β-compact* [2] if every cover of X by β-open subsets has a finite subcover,

(ii) *strongly countably β-compact* if every countable cover of X by β-open subsets has a finite subcover,

(iii) *strongly β-Lindelöf* if every β-open cover of a space X has a countable subcover,

(iv) S-*Lindelöf* [12] if every cover of X by regular closed sets has a countable subcover,

(v) *countably* S-*closed* [5] if every countable cover of X by regular closed sets has a finite subcover,

(vi) S-*closed* [27] if every regular closed cover of X has a finite subcover.

Theorem 4.8. *Let $f : X \to Y$ be a contra almost β-continuous surjection. Then the following statements hold:*

(i) *If X is β-compact, then Y is* S-*closed.*

(ii) *If X is strongly β-Lindelöf, then Y is* S-*Lindelöf,*

(iii) *If X is strongly countably β-compact, then Y is countably* S-*closed.*

Proof. We prove only (i), the proofs of (ii) and (iii) being entirely analogous. Let $\{V_\alpha : \alpha \in I\}$ be any regular closed cover of Y. Since f is almost contra β-continuous, then $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is a β-open cover of X and hence there exists a finite subset I_0 of I such that $X = \bigcup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Therefore, we have $Y = \bigcup\{V_\alpha : \alpha \in I_0\}$ and Y is S-closed. □

Definition 4.6. A space X is said to be

(i) *β-closed* [26] if every cover of X by β-closed sets has a finite subcover,

(ii) *countably β-closed* if every countable cover of X by β-closed sets has a finite subcover,

(iii) *β-Lindelöf* if every cover of X by β-closed sets has a countable subcover,

(iv) *nearly countably compact* [23] if every countable cover of X by regular open sets has a finite subcover,

(v) *nearly compact* [21] if every cover of X by regular open sets has a finite subcover,

(vi) *nearly Lindelöf* [6] if every cover of X by regular open sets has a countable subcover.

Theorem 4.9. *Let $f : X \to Y$ be an contra almost β-continuous surjection. Then the following statements hold:*

(i) *If X is β-closed, then Y is nearly compact.*

(ii) *If X is β-Lindelöf, then Y is nearly Lindelöf.*

(iii) *If X is countably β-closed, then Y is nearly countably compact.*

Proof. (i) Let $\{V_\alpha : \alpha \in I\}$ be any regular open cover of Y. Since f is contra almost β-continuous, then $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is a β-closed cover of X. Since X is β-closed, there exists a finite subset I_0 of I such that $X = \bigcup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Thus we have $Y = \bigcup\{V_\alpha : \alpha \in I_0\}$ and Y is nearly compact.

(ii), (iii) Similar to (i). □

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Department of Mathematics,
Delhi University,
Delhi-110007,
INDIA
sanjaytahiliani@yahoo.com