ANALELE ŞTIINŢIFICE ALE UNIVERSITĂŢII "AL.I. CUZA" DIN IAŞI (S.N.)
MATEMATICĂ, Tomul LVII, 2011, f.2
DOI: 10.2478/v10157-011-0028-x

A COLLOCATION METHOD FOR DIRECT NUMERICAL INTEGRATION OF INITIAL VALUE PROBLEMS IN HIGHER ORDER ORDINARY DIFFERENTIAL EQUATIONS

BY

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Abstract. This paper concerns the solution of initial value problems (IVPs) in ordinary differential equations (ODEs) of orders higher than unity. The Chebyshev polynomials is hereby adopted as basis function in a multi-step collocation technique for the derivation of continuous integration schemes for direct solution of these ODEs without recourse to the conventional approach of first reducing such to their equivalent first order differential systems.

Mathematics Subject Classification 2000: 65L05.

Key words: Chebyshev polynomials, continuous schemes, collocation, approximant, initial value problem.

1. Introduction

We shall consider the class of IVPs in the m-th order ODE:

$$y^{(m)}(x) = f(x, y(x), y^{(1)}(x), y^{(2)}(x), y^{(3)}(x), \cdots, y^{(m-1)}(x)) \tag{1}$$

$$y^{(r)}(a) = \eta_r, r = 0(1)m-1, \tag{2}$$

for $a \leq x \leq b < +\infty$ and where $a, b, \eta_r, r = 0(1)m-1$, are given real constants. The solution of the IVP (1)-(2) will be sought over the discetized interval $[a, b]$:

$$a = x_0 < x_1 < x_2 < x_3 < \cdots < x_{n-2} < x_{n-1} < x_n = b \tag{3}$$

with uniform steplength $h = (b-a)/n$.

Techniques for the integration of the IVP (1)-(2) exist in the literature (see Adeniyi [1], Lambert [9], Onumanyi et al [10], Fatunla [5], Awoyemi et al [4]), some of which involve the reduction into a system of first order problems. However, in this work, we propose direct integration of the IVP (1)-(2) by employing the Chebyshev polynomial (see Fox [6], Gerald [8], Sastry [11]) as basic function in a multi-step collocation procedure, to derive continuous schemes for the class.

In what follows in the next section, we briefly review an antecedent of the procedures as reported by Adeniyi et al [2,3]. Section 3 addresses the central concern of this paper, the derivation of continuous schemes for the IVP (1)-(2), based on Chebyshev polynomials, section 4 presents some numerical examples for illustration and authentication while section 5 finally closes the paper with some concluding remarks.

2. Chebyshev-collocation techniques for first order ODEs

In Adeniyi et al [2,3], we reported that the class of IVPs in first order ODE:

$$y'(x) = f(x, y(x)), a \leq x \leq b < +\infty \tag{4}$$

$$y(a) = y_0 \tag{5}$$

may be solved by seeking an approximant:

$$Y(x) = \sum_{r=0}^{M} a_r T_r(x), 0 < M < +\infty \tag{6}$$

of y(x) which satisfies the equivalent problem:

$$Y'(x) = f(x, Y(x)), x_k \leq x \leq x_{k+p} \tag{7}$$

$$Y(x_k) = Y_k \tag{8}$$

over each sub-interval (3) of [a,b], and where M and p are determined by the class and the step number of the method to be derived. The polynomial $T_r(x)$ in (6) is r-th degree Chebyshev polynomial valid in the interval and it is defined by (see Adeniyi [1])

$$T_r(x) = \cos[n \cos^{-1}(2x - b - a)/(b - a) - 1] \equiv \sum_{k=0}^{r} C_k^r x^r \tag{9}$$

and it satisfies in [a,b], the recurrence relation:

$$T_{r+1}(x) = 2(2x - b - a)/(b - a)T_r(x) - T_{r-1}(x) \quad (10)$$

$$T_0(x) = 1 \quad (11)$$

$$T_1(x) = (2x - b - a)/(b - a). \quad (12)$$

The polynomial $T_r(x)$ performs much better in function approximation problems than the Taylor's polynomial (see Fox and Parker [7]) whose accuracy diminishes as one moves away from the origin. By interpolating (6) and collocating (7) at some appropriately chosen points, a wide range of classes of methods were successfully derived. They included continuous optimal order methods (see Adeniyi and Alabi [2]), predictor-corrector methods, backward differentiation formulae and hybrid schemes. An extension of this collocation method to the class (1)-(2) will now considered in the next section.

3. A Chebyshev-collocation method for higher order ODEs

We consider here the solution of the class of IVP (1)-(2) by seeking the approximant of $y(x)$ of the form (6), that is:

$$Y(x) = \sum_{r=0}^{M} a_r T_r(x), 0 < M < +\infty \quad (13)$$

over each of the sub-inerval $[x_k, x_{k+p}]$ of [a,b] and where,in this case, $M = 2n$ and $p = n$. So then, the problem we are concerned with is:

$$Y^{(m)}(x) = f(x, Y(x), Y^{(1)}(x), Y^{(2)}(x), Y^{(3)}(x), \cdots, Y^{(m-1)}(x)) \quad (14)$$

$$Y(x_k) = Y_k \quad (15)$$

$$Y(x) = \sum_{r=0}^{2n} a_r T_r(x) = \sum_{r=0}^{2n} a_r T_r\left(\frac{2x}{nh} - \frac{2k}{n} - 1\right) \quad (16)$$

over the sub-interval $x_k \leq x \leq x_{k+n}$. Our points of collocation for (14) are x_{k+r}, $r = 0(1)k + n$ and the points of interpolation for (16) are x_{k+r}, $r = 0(1)k + n - 1$. We now proceed from here to consider some specific cases of (14)-(16).

3.1. A two-step method

For an integration scheme of two-step number, we set $n = 2$ in (14)-(16) to have:

$$Y^{(2)}(x) = f(x, Y(x), Y^{(1)}(x)), x_k \leq x \leq x_{k+2} \tag{17}$$

$$Y(x_k) = Y_k \tag{18}$$

$$Y(x) = \sum_{r=0}^{2n} a_r T_r(x) = \sum_{r=0}^{4} a_r T_r(\frac{x}{h} - k - 1). \tag{19}$$

We collocate (17) at $x_{k+r}, r = 0(1)2$ and interpolate (19) at $x_{k+r}, r = 0, 1$ to obtain the linear system:

$$\begin{pmatrix} 1 & -1 & 1 & -1 & 1 \\ 1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 4 & -24 & 80 \\ 0 & 0 & 4 & 0 & -16 \\ 0 & 0 & 4 & 24 & 80 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix} = \begin{pmatrix} Y_k \\ Y_{k+1} \\ h^2 f_k \\ h^2 f_{k+1} \\ h^2 f_{k+2} \end{pmatrix}. \tag{20}$$

We solve (20) for $a_r, r = 0(1)4$, and insert the resulting values in (19) to obtain our proposed continuous integration scheme:

$$\begin{aligned} Y(x) = \frac{h^2}{192} \Bigg\{ & [12\frac{(x-x_k)}{h} - 2\frac{(x-x_k)^3}{h^3} + \frac{(x-x_k)^4}{2h^4}] f_k \\ & + [40\frac{(x-x_k)}{h} + 24\frac{(x-x_k)^3}{h^3} - \frac{(x-x_k)^4}{h^4}] f_{k+1} \\ & + [-4\frac{(x-x_k)}{h} + 2\frac{(x-x_k)^3}{h^3} + \frac{(x-x_k)^4}{2h^4}] f_{k+2} \Bigg\} \\ & - \left\{ \frac{1}{2}\frac{(x-x_k)}{2} Y_k - [2 + \frac{x-x_k}{h}] Y_{k+1} \right\}. \end{aligned} \tag{21}$$

At the grid point x_{k+2}, this reproduces the well-known Numerov's scheme:

$$Y_k - 2Y_{k+1} + Y_{k+2} = \frac{h^2}{12}(f_k + 10 f_{k+1} + f_{k+2}) \tag{22}$$

which is an optimal fourth order method. From (22) we obtain f_{k+2} for our proposed continuous scheme (21).

3.2. A three-step method

By letting $n = 3$ in (14)-(16), we have

$$Y^{(3)}(x) = f(x, Y(x), Y^{(1)}(x), Y^{(2)}(x)), x_k \leq x \leq x_{k+3} \tag{23}$$

$$Y(x_k) = Y_k \tag{24}$$

$$Y(x) = \sum_{r=0}^{2n} a_r T_r(x) = \sum_{r=0}^{6} a_r T_r(\frac{2x}{3h} - \frac{2k}{3} - 1). \tag{25}$$

We collocate (23) at $x_{k+r}, r = 0(1)3$ and interpolate (25) at $x_{k+r}, r = 0, 1, 2$ to obtain the linear system:

$$\begin{pmatrix} 1 & -1 & 1 & -1 & 1 & -1 & 1 \\ 729 & -243 & -567 & 621 & 153 & -723 & 329 \\ 729 & 243 & -567 & -621 & 153 & 723 & 329 \\ 0 & 0 & 0 & 192 & -1536 & 6720 & -24576 \\ 0 & 0 & 0 & 5184 & -13824 & -2880 & 24576 \\ 0 & 0 & 0 & 5184 & 13824 & -2880 & -24576 \\ 0 & 0 & 0 & 192 & 1536 & 6720 & 24576 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{pmatrix} = \begin{pmatrix} Y_k \\ 729Y_{k+1} \\ 729Y_{k+2} \\ 27h^3 f_k \\ 729h^3 f_{k+1} \\ 729h^3 f_{k+2} \\ 729h^3 f_{k+3} \end{pmatrix}.$$

By solving this matrix equation for $a_r, r = 0(1)6$, and inserting the resulting values in (25) we obtain our proposed continuous integration scheme:

$$\begin{aligned} Y(x) = {} & \frac{h^3}{163840}\{[110 - 1024\frac{(x-x_k)}{3h} - 2432\frac{(x-x_k)^2}{9h^2} \\ & + 640\frac{(x-x_k)^3}{27h^3} + 2736\frac{(x-x_k)^4}{81h^4} + 64\frac{(x-x_k)^5}{243h^5} + 2592\frac{(x-x_k)^6}{729h^6}]f_k \\ & + [-9187 - 4096\frac{(x-x_k)}{3h} + 76657\frac{(x-x_k)^2}{9h^2} + 45440\frac{(x-x_k)^3}{27h^3} \\ & + 42768\frac{(x-x_k)^4}{81h^4} - 64\frac{(x-x_k)^5}{243h^5} + 7776\frac{(x-x_k)^6}{729h^6}]f_{k+1} \\ & + [-1328 - 4096\frac{(x-x_k)}{3h} 16044\frac{(x-x_k)^2}{9h^2} + 45440\frac{(x-x_k)^3}{27h^3} \\ & + 42768\frac{(x-x_k)^4}{81h^4} - 64\frac{(x-x_k)^5}{243h^5} - 7776\frac{(x-x_k)^6}{729h^6}]f_{k+2} \\ & + [16 - 1024\frac{(x-x_k)}{3h} - 2788\frac{(x-x_k)^2}{9h^2} + 640\frac{(x-x_k)^3}{27h^3} \\ & - 2736\frac{(x-x_k)^4}{81h^4} + 64\frac{(x-x_k)^5}{243h^5} + 2592\frac{(x-x_k)^6}{729h^6}]f_{k+3}\} \end{aligned} \tag{26}$$

$$+\frac{1}{163840}\{[-20480+18430\frac{(x-x_k)}{9h^2}]Y_k$$
$$+[122880-245760\frac{(x-x_k)}{3h}+184320\frac{(x-x_k)^2}{9h^2}]Y_{k+1}$$
$$+[61440+245760\frac{(x-x_k)}{3h}+184320\frac{(x-x_k)^2}{9h^2}]Y_{k+2}\}.$$

This gives, at the grid point x_{k+3}, the finite difference scheme:

$$3Y_{k+1}-3Y_{k+2}+Y_{k+3}=\frac{h^3}{2}(f_{k+1}+f_{k+2}). \tag{27}$$

3.3. A four-step method

By setting $n = 4$ in (14)-(16), we have the following problem to consider:

$$Y^{(4)}(x)=f(x,Y(x),Y^{(1)}(x),Y^{(2)}(x),Y^{(3)}(x)), x_k\le x\le x_{k+4} \tag{28}$$

$$Y(x_k)=Y_k \tag{29}$$

$$Y(x)=\sum_{r=0}^{2n}a_rT_r(x)=\sum_{r=0}^{8}a_rT_r(\frac{x}{2h}-\frac{k}{2}-1). \tag{30}$$

We collocate (28) at $x_{k+r}, r = 0(1)$ and interpolate (30) at $x_{k+r}, r = 0(1)3$ to obtain the linear system:

$$\begin{pmatrix}1&-1&1&-1&1&-1&1&-1&1\\2&-1&-2&2&-1&-1&2&-1&-1\\1&0&-1&0&1&0&-1&0&1\\2&1&-1&-2&-1&1&2&1&-1\\0&0&0&0&12&-120&648&-2520&7920\\0&0&0&0&12&-60&108&0&-360\\0&0&0&0&12&0&-72&0&240\\0&0&0&0&12&60&108&0&-360\\0&0&0&0&12&120&648&2520&7920\end{pmatrix}\begin{pmatrix}a_0\\a_1\\a_2\\a_3\\a_4\\a_5\\a_6\\a_7\\a_8\end{pmatrix}=\begin{pmatrix}Y_k\\2Y_{k+1}\\Y_{k+2}\\2Y_{k+3}\\h^4f_k\\h^4f_{k+1}\\h^4f_{k+2}\\h^4f_{k+3}\\h^4f_{k+4}\end{pmatrix}.$$

By solving this system for $a_r, r = 0(1)8$, and substituting the resulting values in (30) we obtain our proposed continuous integration scheme:

$$Y(x)=\frac{h^4f_k}{60480}\{55\frac{(x-x_k)}{2h}+11\frac{(x-x_k)^2}{8h^2}+77\frac{(x-x_k)^3}{8h^3}-21\frac{(x-x_k)^5}{16h^5}$$
$$-7\frac{(x-x_k)^6}{64h^6}+3\frac{(x-x_k)^7}{64h^7}+-6\frac{(x-x_k)^8}{1024h^8}\}+\frac{h^4f_{k+1}}{15120}\{-493\frac{(x-x_k)}{2h}$$

$$
\begin{aligned}
&- 53\frac{(x-x_k)^2}{8h^2} + 287\frac{(x-x_k)^3}{8h^3} - 42\frac{(x-x_k)^5}{16h^5} + 7\frac{(x-x_k)^6}{64h^6} \\
&+ 6\frac{(x-x_k)^7}{256h^7} - 6\frac{(x-x_k)^8}{1024h^8}\} + \frac{h^4 f_{k+2}}{10080}\{553\frac{(x-x_k)}{2h} \\
&- 1546\frac{(x-x_k)^2}{16h^2} + 1106\frac{(x-x_k)^3}{16h^3} + 420\frac{(x-x_k)^5}{16h^5} \\
&+ 35\frac{(x-x_k)^6}{64h^6} + 6\frac{(x-x_k)^7}{64h^7} + 6\frac{(x-x_k)^8}{1024h^8}\} \\
&+ \frac{h^4 f_{k+3}}{15120}\{59\frac{(x-x_k)}{2h} - 106\frac{(x-x_k)^2}{16h^2} - 280\frac{(x-x_k)^3}{16h^3} \\
&+ 168\frac{(x-x_k)^5}{64h^5} + 7\frac{(x-x_k)^6}{16h^6} + 6\frac{(x-x_k)^7}{256h^7} \\
&- 6\frac{(x-x_k)^8}{1024h^8}\} + \frac{h^4 f_{k+4}}{60480}\{-82\frac{(x-x_k)}{4h} + 11\frac{(x-x_k)^2}{8h^2} \\
&+ 77\frac{(x-x_k)^3}{8h^3} - 21\frac{(x-x_k)^5}{16h^5} - 7\frac{(x-x_k)^6}{64h^6} + 3\frac{(x-x_k)^7}{64h^7} \\
&+ 6\frac{(x-x_k)^8}{1024h^8}\} + \frac{Y_k}{3}\{\frac{(x-x_k)}{4h} - \frac{(x-x_k)^3}{16h^3}\} - \frac{Y_{k+1}}{4}\{2\frac{(x-x_k)}{h} \\
&- \frac{(x-x_k)^2}{2h^2} - \frac{(x-x_k)^3}{4h^3}\} + Y_{k+2}\{1 + \frac{(x-x_k)}{4h} \\
&- \frac{(x-x_k)^2}{2h^2} - 4h^2 + \frac{(x-x_k)^3}{16h^3}\}.
\end{aligned} \tag{31}
$$

The corresponding finite difference scheme, at the grid point x_{k+4}, is therefore:

$$
\begin{aligned}
&Y_k - 4Y_{k+1} + 6Y_{k+2} - 4Y_{k+3} + Y_{k+4} \\
&\quad = -\frac{h^4}{720}(f_k - 124f_{k+1} - 474f_{k+2} - 124f_{k+3} + f_{k+4})
\end{aligned}
$$

which is of sixth order with error constant $\frac{1}{3024}$.

3.4. A six-step method

For a six-step method, the steps which led to the results of the preceding subsection yielded, at the grid point x_{k+6}, the discrete/finite difference scheme:

$$
\begin{aligned}
&49483Y_{k+6} + 785862Y_{k+5} + 790965Y_{k+4} - 3252620Y_{k+3} \\
&+ 790965Y_{k+2} + 785862Y_{k+1} + 49483Y_k = h^2(1857f_{k+6}
\end{aligned}
$$

$$+110322f_{k+5} + 989739f_{k+4} + 2175924f_{k+3} + 989739f_{k+2} + 110322f_{k+1} + 1857f_k) \tag{32}$$

In a similar vein, Awoyemi et al [4] engaged the monomials x^r to obtain, for a six-step method, the discrete numerical integration scheme:

$$539Y_{k+6} - 4374Y_{k+5} + 32805Y_{k+4} - 57940Y_{k+3} + 32805Y_{k+2} - 4374Y_{k+1} + 539Y_k = h^2(60f_{k+6} + 20040f_{k+3} + 60f_k). \tag{33}$$

Remark. We remark here that, for want of available space, the corresponding continuous schemes for the last two methods, that is, method (32) and method (33) are not included here.

3.5. An analysis of the methods

We provide here an analysis, by way of determining the error and error constants, of our schemes. For illustration, we shall consider this analysis for the method (32).

The linear multi-step method (LMM):

$$\sum_{r=0}^{k} \alpha_r y_{n+r} = h^2 \sum_{r=0}^{k} \beta_r f_{n+r} \tag{34}$$

is said to be of order p if $C_r = 0$ for $r = 0(1)p+1$ but $C_{p+2} \neq 0$, where

$$\begin{aligned} C_0 &= \sum_{r=0}^{k} \alpha_r \\ C_1 &= \sum_{r=0}^{k} r\alpha_r \\ C_2 &= \frac{1}{2!}\sum_{r=0}^{k} r^2\alpha_r - \sum_{r=0}^{k} \beta_r \\ C_q &= \frac{1}{q!}\sum_{r=0}^{k} r^q\alpha_r - \frac{1}{(q-1)!}\sum_{r=0}^{k} r^{q-2}\beta_r. \end{aligned} \tag{35}$$

The quantities C_{p+1} and $C_{p+2}h^{p+2}y^{p+2}(x_n)$ are respectively the error constant and principal truncation error of (34).

Now, for the scheme (32), we have

$$\alpha_0 = 49483, \alpha_1 = 785862, \alpha_2 = 790965, \alpha_3 = -3252620, \alpha_4 = 790965,$$
$$\alpha_5 = 785862, \alpha_6 = 49483, \beta_0 = 1857, \beta_1 = 110322,$$
$$\beta_2 = 989739, \beta_3 = 217592, \beta_4 = 989739, \beta_5 = 110322, \beta_6 = 1857.$$

By employing (35), we obtain $C_r = 0$ for $r = 0(1)9$, and $C_{10} = -0.00048$. Thus the scheme (32) is of order 8 and with an error constant $C_{10} = -0.00048$.

4. Numerical example

We consider here an example for implementation of the algorithm of the schemes (32) and (33), and their corresponding continuous schemes.

Table 1: Errors of Methods for Example 4.1 with $h = 0.1$

x	Cont.scheme f or (33)	Cont. scheme for (32) for (32)	Scheme (32)	Scheme (33)
0.0	0.0	0.0	0.0	0.0
0.1	0.1329867326D-09	0.1708719055D-09		
0.2	0.5872691257D-08	0.6836010114D-08		
0.3	0.1327845616D-07	0.1555757709D-07		
0.4	0.2317829012D-07	0.2880198295D-07		
0.5	0.3218793564D-07	0.4802328029D-07		
0.6	0.6871246012D-07	0.7628531256D-07		
0.7	0.1012728156D-06	0.1157914170D-06		
0.8	0.1231093271D-06	0.1727046080D-06		
0.9	0.2019286712D-06	0.2561456831D-06		
1.0	0.2990871645D-06	0.3815695118D-06	0.2990871645D-06	0.3815695118D-06

Example 4.1.

$$y^{(2)}(x) = x(y^(2))^2, \quad 0 \le x \le 1$$
$$y(0) = 1$$
$$y^{(1)}(0) = \frac{1}{2}.$$

The analytical solution is

$$y(x) = 1 + \frac{1}{2}\ln\left(\frac{2+x}{2-x}\right).$$

See Table 1 for some computed results.

5. Conclusion

A continuous formulation of some finite difference numerical schemes for direct integration of initial value problems in ODEs of higher order than unity and for which the Chebyshev polynomials was employed as the basis function in a multi-step collocation approach has been presented.

The circumvention of the usual first step of reducing the ODEs to systems of first order equations before the actual process of solution makes the derived schemes desirable. The efficiency of these schemes, derived from their ability to yield several output of solutions at the off-grid points without requiring additional interpolation and at no extra cost, renders them attractive for application for higher order ODEs.

We also note that our derived scheme, when tested on a problem performed better than the scheme (33) in terms of accuracy, thus supporting the theoretical smaller error constant.

Acknowlegement. The first author thankfully acknowledgements financial support rom the University of Ilorin Senate Research Grant.

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Received: 2.III.2009

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