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UNIQUENESS OF ENTIRE FUNCTIONS*

BY

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Abstract. In this paper, we study the uniqueness problems on meromorphic functions sharing a finite set. The results extend and improve some theorems obtained earlier by Fang (2002) and Zhang-Lin (2008).

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1. Introduction and results

In this paper, we will use the standard notations of Nevanlinna's value distribution theory (cf. [2], [5]).

Let f be a nonconstant meromorphic function in the whole complex plane $\mathbf{C}$, we set $E(a,f) = \{z|f(z)-a=0,\text{counting multiplicties}\}$. In general, put $E(S,f) = \bigcup_{a\in S} E(a,f)$, where S denotes a set of complex numbers. Let k be a positive integer. Set

$$E_k(S,f) = \bigcup_{a\in S}\{z|f(z)-a=0, \exists\ i, 0<i\le k,\ \text{s.t.}\ f^{(i)}(z)\neq 0\},$$

where each zero of $f(z)-a$ with multiplicity m counted m times when $m \le k$ in $E(S,f)$.

Let f and g be two nonconstant entire functions, n, m, l, t and p be positive integers, we set

$$F = [f^n(f^l-1)^t]^{(p)}, G = [g^n(g^l-1)^t]^{(p)}, \tag{1.1}$$

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$$H_m = \frac{(F^m)''}{(F^m)'} - 2\frac{(F^m)'}{F^m - 1} - \frac{(G^m)''}{(G^m)'} + 2\frac{(G^m)'}{G^m - 1} \tag{1.2}$$

and $S_m = \{1, \omega, \omega^2, \cdots, \omega^{m-1}\}$, where $\omega = e^{\frac{2\pi}{m}i}$.

In 2002, Fang proved the following result.

Theorem A ([1]). *Let f and g be two nonconstant entire functions, and let n, k be tow positive integers with $n > 2k + 8$. If $[f^n(z)(f(z) - 1)]^{(k)}$ and $[g^n(z)(g(z) - 1)]^{(k)}$ share 1 CM, then $f(z) \equiv g(z)$.*

In a latter paper, Zhang and Lin improved Theorem A and obtained the following result.

Theorem B ([4]). *Let f and g be two nonconstant entire functions, and let n, m and k be three positive integer with $n > 2k+m+4$. If $[f^n(z)(f(z)-1)^m]^{(k)}$ and $[g^n(z)(g(z) - 1)^m]^{(k)}$ share 1 CM, then $f(z) \equiv g(z)$ or f and g satisfy the algebraic equation $R(f, g) \equiv 0$, where $R(\omega_1, \omega_2) = \omega_1^n(\omega_1 - 1)^m - \omega_2^n(\omega_2 - 1)^m$.*

In this article, we prove

Theorem 1. *Let f and g be two transcendental entire functions, n, m, t, l, p be positive integers. If $E_1(S_m, [f^n(f^l - 1)^t]^{(p)}) = E_1(S_m, [g^n(g^l - 1)^t]^{(p)})$ and $n > \frac{6}{m} + 3tl + 4p$, then $f(z) \equiv bg(z)$, where $b^l = 1$.*

2. Lemmas

To prove the theorem, we need the following lemmas.

Lemma 1 ([3]). *Let $f(z)$ be a nonconstant meromorphic function, k be a positive integer, if $f^{(k)} \not\equiv 0$, then $N(r, \frac{1}{f^{(k)}}) \le N(r, \frac{1}{f}) + k\overline{N}(r, f) + S(r, f)$.*

Lemma 2. *Let F, G be defined as (1.1) and (1.2). If $E_1(S_m, F) = E_1(S_m, G)$, and $n > \frac{6}{m} + 3tl + 4p$, then $H_m \equiv 0$.*

Proof. If $H_m \not\equiv 0$, then $E_1(1, F^m) = E_1(1, G^m)$, since $E_1(S_m, F) = E_1(S_m, G)$. Suppose that z_0 is a common simple zero point of $F^m - 1$ and $G^m - 1$, then it follows from (1.2) that z_0 is a zero point of H_m, and zero point of F^m or G^m with multiplicity 1 also are not poles of H_m. Thus, we have

$$N_{1)}\left(r,\frac{1}{F^m-1}\right)=N_{1)}\left(r,\frac{1}{G^m-1}\right)\le N\left(r,\frac{1}{H_m}\right)\le T(r,H_m)+O(1)$$
$$\le N(r,H_m)+S(r).$$

By the definition of H_m, we have poles of H_m with multiplicity 1. Thus

$$\begin{aligned}N_{1)}\left(r,\frac{1}{F^m-1}\right)=N_{1)}\left(r,\frac{1}{G^m-1}\right)&\le \overline{N}_{(2}\left(r,\frac{1}{F^m}\right)+\overline{N}_{(2}\left(r,\frac{1}{G^m}\right)\\&+\overline{N}_0\left(r,\frac{1}{(F^m)'}\right)+\overline{N}_0\left(r,\frac{1}{(G^m)'}\right)+\overline{N}_{(2}\left(r,\frac{1}{F^m-1}\right)\\&+\overline{N}_{(2}\left(r,\frac{1}{G^m-1}\right)+S(r).\end{aligned}\tag{2.1}$$

Where $S(r)=\max\{S(r,f),S(r,g)\}$.

By the second fundamental theorem, we have

$$\begin{aligned}T(r,F^m)+T(r,G^m)&\le \overline{N}\left(r,\frac{1}{F^m}\right)+\overline{N}\left(r,\frac{1}{F^m-1}\right)+\overline{N}\left(r,\frac{1}{G^m}\right)\\&+\overline{N}\left(r,\frac{1}{G^m-1}\right)-\left[N_0\left(r,\frac{1}{(F^m)'}\right)+N_0\left(r,\frac{1}{(G^m)'}\right)\right]+S(r).\end{aligned}\tag{2.2}$$

By Lemma 1, we get $N(r,\frac{1}{(G^m)'})\le N(r,\frac{1}{G^m})+S(r)$. Thus

$$\overline{N}_0\left(r,\frac{1}{(G^m)'}\right)+\overline{N}_{(2}\left(r,\frac{1}{G^m-1}\right)+N_{(2}\left(r,\frac{1}{G^m}\right)-\overline{N}_{(2}\left(r,\frac{1}{G^m}\right)$$
$$\le N\left(r,\frac{1}{(G^m)'}\right)\le N\left(r,\frac{1}{G^m}\right)+S(r).$$

It follows that

$$\overline{N}_0\left(r,\frac{1}{(G^m)'}\right)+\overline{N}_{(2}\left(r,\frac{1}{G^m-1}\right)\le \overline{N}\left(r,\frac{1}{G^m}\right)+S(r).\tag{2.3}$$

Similarly, we have

$$\overline{N}_0\left(r,\frac{1}{(F^m)'}\right)+\overline{N}_{(2}\left(r,\frac{1}{F^m-1}\right)\le \overline{N}\left(r,\frac{1}{F^m}\right)+S(r,f).\tag{2.4}$$

From (2.1)-(2.4) we have

$$
\begin{aligned}
m(T(r,F)+T(r,G)) &\leq \overline{N}\left(r,\frac{1}{F^m}\right)+\overline{N}_{1)}\left(r,\frac{1}{F^m-1}\right)\\
&+\overline{N}_{(2}\left(r,\frac{1}{F^m-1}\right)+\overline{N}\left(r,\frac{1}{G^m}\right)+\overline{N}_{1)}\left(r,\frac{1}{G^m-1}\right)\\
&+\overline{N}_{(2}\left(r,\frac{1}{G^m-1}\right)-\left[N_0\left(r,\frac{1}{(F^m)'}\right)+N_0\left(r,\frac{1}{(G^m)'}\right)\right]\\
&+S(r)\leq 4\overline{N}\left(r,\frac{1}{F^m}\right)+4\overline{N}\left(r,\frac{1}{G^m}\right)+2\overline{N}_{(2}\left(r,\frac{1}{F^m}\right)\\
&+2\overline{N}_{(2}\left(r,\frac{1}{G^m}\right)+S(r).
\end{aligned}\tag{2.5}
$$

Since

$$
\begin{aligned}
\overline{N}\left(r,\frac{1}{F^m}\right)+\overline{N}_{(2}\left(r,\frac{1}{F^m}\right)&\leq N\left(r,\frac{1}{F^m}\right)\\
&-\left[N_{(3}\left(r,\frac{1}{F^m}\right)-2\overline{N}_{(3}\left(r,\frac{1}{F^m}\right)\right],
\end{aligned}
$$

and

$$
N_{(3}\left(r,\frac{1}{F^m}\right)-2\overline{N}_{(3}\left(r,\frac{1}{F^m}\right)\geq[m(n-p)-2]N\left(r,\frac{1}{f}\right),
$$

we have

$$
\begin{aligned}
&\overline{N}\left(r,\frac{1}{F^m}\right)+\overline{N}_{(2}\left(r,\frac{1}{F^m}\right)\\
&\leq N\left(r,\frac{1}{F^m}\right)-[m(n-p)-2]N\left(r,\frac{1}{f}\right).
\end{aligned}\tag{2.6}
$$

Similarly,

$$
\begin{aligned}
&\overline{N}\left(r,\frac{1}{G^m}\right)+\overline{N}_{(2}\left(r,\frac{1}{G^m}\right)\\
&\leq N\left(r,\frac{1}{G^m}\right)-[m(n-p)-2]N\left(r,\frac{1}{g}\right).
\end{aligned}\tag{2.7}
$$

Combining (2.5)-(2.7), we have

$$m[T(r,F)+T(r,G)] \le 2\overline{N}\left(r,\frac{1}{F^m}\right)+2\overline{N}\left(r,\frac{1}{G^m}\right)$$
$$+2\left[N\left(r,\frac{1}{F^m}\right)-(m(n-p)-2)N\left(r,\frac{1}{f}\right)\right]$$
$$+2\left[N\left(r,\frac{1}{G^m}\right)-(m(n-p)-2)N\left(r,\frac{1}{g}\right)\right]+S(r)$$
$$\le 2\overline{N}\left(r,\frac{1}{F^m}\right)+2\overline{N}\left(r,\frac{1}{G^m}\right)+2N\left(r,\frac{1}{F^m}\right)+2N\left(r,\frac{1}{G^m}\right)$$
$$-2(m(n-p)-2)\left[N\left(r,\frac{1}{f}\right)+N\left(r,\frac{1}{g}\right)\right]+S(r)=4N\left(r,\frac{1}{F^m}\right)$$
$$+4N\left(r,\frac{1}{G^m}\right)-[4m(n-p)-6)\left[N\left(r,\frac{1}{f}\right)+N\left(r,\frac{1}{g}\right)\right]+S(r).$$

By Lemma 1, we have

$$m\left(m\left(r,\frac{1}{F_1}\right)+m\left(r,\frac{1}{G_1}\right)\right)\le m\left(m\left(r,\frac{1}{F}\right)+m\left(r,\frac{1}{G}\right)\right)+S(r)$$
$$\le 3m\left[N\left(r,\frac{1}{F}\right)+N\left(r,\frac{1}{G}\right)\right]-[4m(n-p)-6)\left[N\left(r,\frac{1}{f}\right)+N\left(r,\frac{1}{g}\right)\right]$$
$$+S(r)\le 3m\left[N\left(r,\frac{1}{F_1}\right)+N\left(r,\frac{1}{G_1}\right)\right]$$
$$-[4m(n-p)-6)\left[N\left(r,\frac{1}{f}\right)+N\left(r,\frac{1}{g}\right)\right]+S(r),$$

where $F_1=f^n(f^l-1)^t, G_1=g^n(g^l-1)^t$. It follows that

$$m[T(r,F_1)+T(r,G_1)]\le 4m\left[N\left(r,\frac{1}{F_1}\right)+N\left(r,\frac{1}{G_1}\right)\right]$$
$$-[4m(n-p)-6)\left[N\left(r,\frac{1}{f}\right)+N\left(r,\frac{1}{g}\right)\right]+S(r).$$

We get $[m(n+tl)-4mtl-4mp-6][T(r,f)+T(r,g)]\le S(r)$, which contradicts the assumption that $n>\frac{6}{m}+3tl+4p$. Therefore $H_m\equiv 0$, which completes the proof of Lemma 2.

Lemma 3. *Let f be a transcendental meromorphic functions, a_1 and a_2 be two meromorphic functions such that $T(r,a_j)=S(r,f)(j=1,2)$ and*

$a_1 \not\equiv a_2$, then

$$T(r,f) \le \overline{N}(r,f) + \overline{N}\left(r, \frac{1}{f-a_1}\right) + \overline{N}\left(r, \frac{1}{f-a_2}\right) + S(r,f).$$

Lemma 4. *Let f be a transcendental entire function, k be a positive integer, and c be a nonzero finite complex number. Then*

$$\begin{aligned} T(r,f) &\le N\left(r,\frac{1}{f}\right) + N\left(r,\frac{1}{f^{(k)}-c}\right) - N\left(r,\frac{1}{f^{(k+1)}}\right) + S(r,f) \\ &\le N_{k+1}\left(r,\frac{1}{f}\right) + \overline{N}\left(r,\frac{1}{f^{(k)}-c}\right) - N_0\left(r,\frac{1}{f^{(k+1)}}\right) + S(r,f), \end{aligned}$$

where $N_0(r, 1/f^{(k+1)})$ is the counting function which only counts those points such that $f^{k+1)} = 0$ but $f(f^k - c) \neq 0$.

3. Proof of Theorem 1

Let F, G and defined as (1.1) and (1.2).

By Lemma 2, we have $H_m \equiv 0$, that is

$$\frac{(F^m)''}{(F^m)'} - 2\frac{(F^m)'}{F^m-1} \equiv \frac{(G^m)''}{(G^m)'} - 2\frac{(G^m)'}{G^m-1}.$$

Thus

$$\frac{1}{G^m-1} \equiv \frac{A}{F^m-1} + B, \tag{3.1}$$

where $A \neq 0$ and B are two constants. Hence $E(1,F^m) = E(1,G^m)$, $T(r,F) = T(r,G)$.

(I) Now we claim that

$$f^n(f^l-1)^t \equiv ag^n(g^l-1)^t. \tag{3.2}$$

Next we consider the following two cases.

Case 1. When $B = 0$, by (3.1), we have

$$F^m = AG^m + (1-A). \tag{3.3}$$

(a) If $A = 1$, then by (3.3), we have $F^m = G^m$, and hence $f^n(f^l-1)^t \equiv ag^n(g^l-1)^t$.

(b) If $A \neq 1$, then by (3.3), we have

$$F^{m-1}F' = AG^{m-1}G'. \tag{3.4}$$

From (3.3) and (3.4) we get: when $F = 0$, $G^m \neq 0, 1$ and $G' = 0$, when $G = 0$, $F^m \neq 0, 1$ and $F' = 0$. Thus

$$\begin{aligned} N\left(r, \frac{1}{F}\right) - N_0\left(r, \frac{1}{(G^m)'}\right) &= S(r, F), \\ N\left(r, \frac{1}{G}\right) - N_0\left(r, \frac{1}{(F^m)'}\right) &= S(r, F). \end{aligned} \tag{3.5}$$

By the second fundamental theorem, we have

$$\begin{aligned} T(r, F^m) &\leq \overline{N}\left(r, \frac{1}{F^m}\right) + \overline{N}\left(r, \frac{1}{F^m - (1-A)}\right) - N_0\left(r, \frac{1}{(F^m)'}\right) + S(r, F) \\ &\leq \overline{N}\left(r, \frac{1}{F}\right) + \overline{N}\left(r, \frac{1}{G}\right) - N_0\left(r, \frac{1}{(F^m)'}\right) + S(r, F). \end{aligned} \tag{3.6}$$

Similarly, we have

$$T(r, G^m) \leq \overline{N}\left(r, \frac{1}{G}\right) + \overline{N}\left(r, \frac{1}{F}\right) - N_0\left(r, \frac{1}{(G^m)'}\right) + S(r, G). \tag{3.7}$$

From (3.5)-(3.7), we have

$$2mT(r, F) \leq \left[\overline{N}\left(r, \frac{1}{G}\right) + \overline{N}\left(r, \frac{1}{F}\right)\right] + S(r, F) \leq 2T(r, F) + S(r, F).$$

Hence $m = 1$. By (3.3) we get

$$f^n(f^l - 1)^t \equiv ag^n(g^l - 1)^t + P(z), \tag{3.8}$$

where $P(z)$ is a polynomial of degree at most $p - 1$.

If $P(z) \not\equiv 0$, by (3.8) and Lemma 3, we have

$$\begin{aligned} T(r, f^n(f^l - 1)^t) &\leq \overline{N}(r, f^n(f^l - 1)^t) + \overline{N}\left(r, \frac{1}{f^n(f^l - 1)^t}\right) \\ &+ \overline{N}\left(r, \frac{1}{f^n(f^l - 1)^t - P}\right) + S(r, f) \leq \overline{N}\left(r, \frac{1}{f}) + \overline{N}(r, \frac{1}{f^l - 1}\right) \\ &+ \overline{N}\left(r, \frac{1}{g}\right) + \overline{N}\left(r, \frac{1}{g^l - 1}\right) + S(r, f) \leq 2(1 + l)T(r, f) + S(r, f). \end{aligned}$$

Thus, $n + tl \leq 2(1 + l)$, which contradicts the assumption that $n > \frac{6}{m} + 3tl + 4p$.

Case 2. When $B \neq 0$, by (3.1), we have

$$\frac{1}{G^m - 1} = B\frac{F^m + (\frac{A}{B} - 1)}{F^m - 1}, \quad \frac{A}{F^m - 1} = -B\frac{G^m - (\frac{1}{B} + 1)}{G^m - 1}, \tag{3.9}$$

and

$$\frac{G^{m-1}G'}{(G^m - 1)^2} = A\frac{F^{m-1}F'}{(F^m - 1)^2}.$$

Thus

$$F^m + \left(\frac{A}{B} - 1\right) \neq 0, \quad G^m - \left(\frac{1}{B} + 1\right) \neq 0. \tag{3.10}$$

(a) If $A = B$.

By (3.9), we have $F \neq 0$. Since $F = (f^n(f^l - 1)^t)^{(p)}$ and $n > p$, thus $f \neq 0$. Let $f = e^\alpha$, where α is a nonconstant entire function. Thus

$$f^n(f^l - 1)^t = e^{n\alpha}\sum_{j=0}^{t}(-1)^{t-j}C_t^j e^{lj\alpha} = \sum_{j=0}^{t}(-1)^{t-j}C_t^j e^{(n+lj)\alpha}.$$

Let

$$((-1)^{t-j}C_t^j e^{(n+lj)\alpha})^{(p)} = P_j(\alpha', \alpha'', \cdots, \alpha^{(p)})e^{(n+lj)\alpha},$$

where $P_j(\alpha', \alpha'', \cdots, \alpha^{(p)})(j = 0, 1, 2, \cdots, t)$ are differential polynomials. Thus

$$F = \sum_{j=0}^{t} P_j(\alpha', \alpha'', \cdots, \alpha^{(p)})e^{(n+lj)\alpha} = e^{n\alpha}\sum_{j=0}^{t} P_j(\alpha', \alpha'', \cdots, \alpha^{(p)})e^{lj\alpha}$$
$$= e^{n\alpha}F_0,$$

where $F_0 = \sum_{j=0}^{t} P_j(\alpha', \alpha'', \cdots, \alpha^{(p)})e^{lj\alpha}$.

Obviously, there exists $j(0 \leq j \leq t)$, such that $P_j(\alpha', \alpha'', \cdots, \alpha^{(p)}) \not\equiv 0$. Suppose $P_0(\alpha', \alpha'', \cdots, \alpha^{(p)}) \not\equiv 0$. Since $F \neq 0$, thus $F_0 \neq 0$. Since f is a nonconstant entire function we use Lemma 3 to obtain

$$\begin{aligned} ltT(r,e^{\alpha}) &= T(r,F_0) \le \overline{N}\left(r,\frac{1}{F_0}\right) \\ &+ \overline{N}\left(r,\frac{1}{F_0 - P_0(\alpha',\alpha'',\cdots,\alpha^{(p)})}\right) + \overline{N}\left(r,F_0\right) + S(r,e^{\alpha}) \\ &= \overline{N}\left(r,\frac{1}{\sum_{j=1}^{t} P_j(\alpha',\alpha'',\cdots,\alpha^{(p)})e^{lj\alpha}}\right) + S(r,e^{\alpha}) \\ &= \overline{N}\left(r,\frac{1}{\sum_{j=1}^{t} P_j(\alpha',\alpha'',\cdots,\alpha^{(p)})e^{l(j-1)\alpha}}\right) + S(r,e^{\alpha}) \\ &\le l(t-1)T(r,e^{\alpha}) + S(r,e^{\alpha}), \end{aligned}$$

which is a contradiction.

(b) If $A \neq B$ and $B = -1$.

By (3.9), we have $G \neq 0$. Since $G = (g^n(g^l-1)^t)^{(p)}$ and $n > p$, thus $g \neq 0$. Let $g = e^{\beta}$, where β is a nonconstant entire function. Similarly, we have $ltT(r,e^{\beta}) \le l(t-1)T(r,e^{\beta}) + S(r,e^{\beta})$, which is a contradiction.

(c) If $A \neq B$ and $B \neq -1$.

When $m > 1$, by (3.10) and the second fundamental theorem, we have

$$\begin{aligned} T(r,G^m) &\le \overline{N}(r,\frac{1}{G^m}) + \overline{N}(r,\frac{1}{G^m - (\frac{1}{B}+1)}) + \overline{N}(r,G^m) + S(r,G) \\ &\le \overline{N}(r,\frac{1}{G}) + S(r,G) \le T(r,G) + S(r), \end{aligned}$$

thus G is constant, hence g is constant, which is a contradiction.

When $m = 1$, by (3.10), we have $F + (\frac{A}{B} - 1) \neq 0$, thus $(f^n(f^l-1)^t)^{(p)} + (\frac{A}{B} - 1) \neq 0$, by Lemma 4, we have

$$\begin{aligned} &(n+lt)T(r,f) = T(r,f^n(f^l-1)^t) + S(r,f) \le N_{p+1}\left(r,\frac{1}{f^n(f^l-1)^t}\right) \\ &+ \overline{N}\left(r,\frac{1}{(f^n(f^l-1)^t)^{(p)} + (\frac{A}{B}-1)}) - N_0(r,\frac{1}{(f^n(f^l-1)^t)^{(p+1)}}\right) + S(r,f) \\ &\le N_{p+1}\left(r,\frac{1}{f^n(f^l-1)^t}\right) + S(r,f) \le (p+1)\overline{N}\left(r,\frac{1}{f}\right) \\ &+ N_{p+1}\left(r,\frac{1}{(f^l-1)^t}\right) + S(r,f) \le (p+1+lt)T(r,f) + S(r,f), \end{aligned}$$

thus $n \le p+1$, which contradicts the assumption that $n > \frac{6}{m} + 3tl + 4p$.

By case 1 and case 2, we get (3.2).

By (3.2), we have

$$\begin{aligned} & f^{n-1}(f^l-1)^{t-1}\left(f^l - \frac{n}{n+tl}\right) f' \\ & = a g^{n-1}(g^l-1)^{t-1}\left(g^l - \frac{n}{n+tl}\right) g'. \end{aligned} \tag{3.11}$$

From (3.2) and (3.11), we get

(i) When $f=0$, have $g=0$ or $g^l=1$.

(ii) When $f^l=1$, have $g^l=1$ or $g=0$.

(iii) When $f^l = \frac{n}{n+tl}$, have $g^l = \frac{n}{n+tl}$ or $g'=0$ (such that $g^l \ne \frac{n}{n+tl}$, $g \ne 0$, $g^l \ne 1$).

By (3.2), (i) and (ii), we have

$$\overline{N}\left(r, \frac{1}{f^l-1}\right) - \overline{N}\left(r, \frac{1}{f^l-1}, \frac{1}{g^l-1}\right) \le \frac{t}{n} N\left(r, \frac{1}{f^l-1}\right), \tag{3.12}$$

and

$$\overline{N}\left(r, \frac{1}{f}\right) - \overline{N}\left(r, \frac{1}{f}, \frac{1}{g}\right) \le \frac{t}{n} N\left(r, \frac{1}{g^l-1}\right). \tag{3.13}$$

Using the fact that f and g are nonconstant entire functions and the second fundamental theorem, we have

$$\begin{aligned} 2lT(r,f) \le & \overline{N}\left(r, \frac{1}{f}\right) + \overline{N}\left(r, \frac{1}{f^l-1}\right) + \overline{N}\left(r, \frac{1}{f^l - \frac{n}{n+tl}}\right) \\ & - N_0\left(r, \frac{1}{f'}\right) + S(r,f) \end{aligned} \tag{3.14}$$

and

$$\begin{aligned} 2lT(r,g) \le & \overline{N}\left(r, \frac{1}{g}\right) + \overline{N}\left(r, \frac{1}{g^l-1}\right) + \overline{N}\left(r, \frac{1}{g^l - \frac{n}{n+tl}}\right) \\ & - N_0\left(r, \frac{1}{g'}\right) + S(r,g). \end{aligned} \tag{3.15}$$

If $f^l \equiv g^l$, then there exists constant b, such that $f \equiv bg$, where $b^l = 1$.

If $f^l \not\equiv g^l$, by (3.12)-(3.15),(i)-(iii), we have

$$
\begin{aligned}
4lT(r,f) = 2l[T(r,f)+T(r,g)] \le 2\overline{N}\left(r,\frac{1}{f},\frac{1}{g}\right) \\
+ 2\overline{N}\left(r,\frac{1}{f^l-1},\frac{1}{g^l-1}\right) + 2\overline{N}\left(r,\frac{1}{f^l-\frac{n}{n+tl}},\frac{1}{g^l-\frac{n}{n+tl}}\right) \\
+ \frac{2t}{n}N\left(r,\frac{1}{f^l-1}\right) + \frac{2t}{n}N\left(r,\frac{1}{g^l-1}\right) + S(r,f) \\
\le 2N\left(r,\frac{1}{f^l-V^l}\right) + \frac{2t}{n}N\left(r,\frac{1}{f^l-1}\right) \\
+ \frac{2t}{n}N\left(r,\frac{1}{g^l-1}\right) + S(r,g) \le \left(2l+\frac{4tl}{n}\right)T(r,f) + S(r,f).
\end{aligned}
$$

Thus $2l \le \frac{4tl}{n}$, which contradicts the assumption that $n > \frac{6}{m} + 3tl + 4p$.

Summarizing the above discussion we obtain Theorem 1.

REFERENCES

1. Fang, M.-L. – *Uniqueness and value-sharing of entire functions,* Comput. Math. Appl., 44 (2002), 823–831.
2. Hayman, W.K. – *Meromorphic Functions,* Oxford Mathematical Monographs Clarendon Press, Oxford, 1964.
3. Jiangtao, LLi; Chunghong, Li – *Uniqueness of meromorphic or entire functions and their differential polyomials,* Acta Math.Scientia, 26A (2006), 1166–1178.
4. Zhang, X.-Y.; Lin, W.-C. – *Uniqueness and value-sharing of entire functions,* J. Math. Anal. Appl., 343 (2008), 938–950.
5. Yang, Lo – *Value Distribution Theory,* Springer-Verlag, Berlin, Science Press Beijing, Beijing, 1993.

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