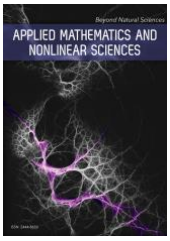
sciendo Applied Mathematics and Nonlinear Sciences, 9(1) (2024) 1-17



Applied Mathematics and Nonlinear Sciences

https://www.sciendo.com

Optimization strategy of mixed-integer linear planning in logistics distribution

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Submission Info

Communicated by Z. Sabir
Received April 4, 2024
Accepted June 28, 2024
Available online August 5, 2024

Abstract

The evolution of the logistics and distribution industry, notably the express delivery sector, has significantly increased its prevalence in everyday life. This escalation necessitates an ongoing innovation in logistics strategies and enhancements in service quality, positioning these elements at the heart of industry focus. This study initially addresses the vehicular route planning issue within logistics distribution, selecting Company A as a case study to examine inherent logistical challenges. Subsequently, it develops an optimal route planning model under time constraints. It is converted into a mixed-integer linear programming model through techniques such as variable substitution and segmented linear approximation. This conversion facilitates rapid solutions by mathematical planning solvers. The research contrasts the logistics and distribution performance before and after optimization to assess the efficacy of the proposed strategies. Findings indicate that optimized vehicular route planning achieves a more equitable distribution of delivery tasks, along with substantial improvements in the complexity of transportation routes and the reduction of travel distances. Specifically, the optimization results in a significant decrease in the penalty costs associated with the vehicles, with the penalty cost for the third vehicle involved in the study reduced by 66.40%. This evidence supports the implementation of scientific vehicle scheduling strategies by logistics firms, underscoring the model's practical implications.

Keywords: Mixed integer linear programming; Path planning; Time constraint; Logistics distribution.
AMS 2010 codes: 03B70

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ISSN 2444-8656
https://doi.org/10.2478/amns-2024-1904

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1 Introduction

Along with China's accession to the WTO and the development of the economic globalization process, Chinese enterprises are facing opportunities and challenges. Logistics customer service will be the enterprise in the future to form a competitive advantage. Good logistics customer service level is the result of the optimization design of the overall logistics system of the enterprise, and the optimization design of the logistics system can not be separated from the application of linear programming methods [1-3]. Facing the new situation, the high-quality development of modern logistics is related to the safe and stable operation of the supply chain, which is an important part of the smooth national economic cycle [4]. With the rapid development of big data, the Internet, e-commerce, and artificial intelligence, the modern logistics system is also constantly changing. Logistics demand and supply capacity are steadily improving, and the delivery efficiency, service quality, cost, and expense of logistics have put forward higher requirements, forcing the transformation and upgrading of logistics to enter the stage of high-quality development in pursuit of high-quality, high-efficiency, high-efficiency and high-yield [5-7].

Logistics distribution is a very important link in supply chain management. Excessive logistics cost consumption makes social resources too wasteful, thus affecting the development of the national economy [8]. Extensive and in-depth research on logistics distribution problems can not only enrich the content of the discipline of operations research but also apply its research results in the field of modern logistics distribution and transportation management to improve the operational efficiency of logistics and reduce transportation costs, which has great theoretical and practical research value [9-11]. Has been about logistics distribution system planning problem research is a research topic worthy of attention. The current research on the layout of logistics distribution systems is still immature. The development trend of energization of logistics distribution systems and the lack of scientific and effective methods for the cooperation of logistics enterprises highlight the importance of logistics distribution system planning research [12-14].

The most direct value of optimizing logistics and distribution for the industry and each of its practitioners is to reduce costs and improve efficiency. Wang, Y et al. constructed a two-level collaborative logistics joint distribution model to improve the efficiency of the whole logistics system, proposed a novel linear optimization method to minimize the total cost, and used a genetic algorithm and improved Shapley's value theory to obtain a better decision, which guarantees a balanced resource allocation both globally and locally [15]. Rodríguez, J. V et al. pointed out that logistics management can optimize the distribution of goods through the supply chain through a variety of strategies, emphasizing the role played by geographic information systems (GIS) in the collection, processing and analysis of location-related data, and using this as a basis for constructing a cost-effective model (GUSEK) for determining the location of logistics centers and the type of vehicles [16]. Pan, J. S et al. were inspired by the predatory behavior of A. grandiflora, proposed a novel intelligent evolutionary algorithm, and verified the feasibility of the proposed algorithm by empirical analysis using the problem of site selection of logistics and distribution centers as an example [17]. Mejjaouli, S et al. discussed the complexity of the transportation of perishable goods and how the challenges can be addressed through cold chain supply chain and logistics management, proposing the use of cloud computing models and virtual machine technologies (VMs) to support the decision-making process for each flight or sea shipment and to enhance the regulation of goods along their transmission paths globally through real-time sharing of critical parameters such as temperature data, location information, etc. [18].

Integer linear programming can be a good solution to the vehicle path problem in logistics and distribution, Sangaiah, A. K et al. illustrated the process of conversion from gasoline to energy. They emphasized the importance of supply and management planning for the gradual increase in the

demand for liquefied natural gas (LNG), mentioning the reduction of costs and depreciation of market volatility risks through effective management and how the use of the complex optimization software GAMS and the COA, a demographic-based optimization algorithm, to solve the related problems [19]. Schiffer, M et al. discussed a robust vehicle route planning approach considering multiple factors such as uncertain customer patterns, demand, and service time windows, and used a parallelized approximation algorithm to address large-scale instances as well as the presence of high computational effort in neighborhood search [20]. Li, Y et al. presented a multi-objective optimization model that integrates the objectives of production planning, inventory, and transportation cost minimization with service level maximization and developed a dual-objective mixed-integer linear programming solution combining an iterative heuristic algorithm and fuzzy logic approach for this problem [21].

In this paper, the vehicle path problem in logistics and distribution is described visually through the vehicle path problem schematic, and the current situation and issues of the logistics and distribution business of Company A are explored according to the operation flow of Company A's logistics and distribution business. Based on the common methods of optimizing the vehicle path problem, this paper optimizes the vehicle path planning model using the mixed integer linear programming method. New decision variables are introduced to replace the variables. Segmented linear functions are used, segmented straight lines are selected using the tangent method, and straight lines are merged using the multiple choice model to finally linearize the original vehicle path problem model into a new mixed-integer linearized model. The optimized model's role in improving logistics and distribution services' quality and reducing costs has been confirmed through example analysis.

2 Vehicle Path Problems in Logistics and Distribution

2.1 Logistics and Distribution

In the whole logistics work, the distribution service belongs to the last link. At the same time, the link is also direct contact with the customer, and the link between the customer is the closest, so the link directly affects the level of customer satisfaction, and customer satisfaction also greatly affects the development of distribution enterprises. In short, the distribution of economic growth of enterprises is a particularly important link. The cost of this link in overall logistics and transportation costs accounts for a large proportion. In the era of information technology, the role and function of distribution have been further enriched. For example, in the process of distribution, the use of big data technology for the integration of customer data, and with the help of data analysis to explore the economic value of the information behind the financial benefits of distribution enterprises to assist.

2.2 Vehicle Path Problem and Optimization

2.2.1 Vehicle path problems

The Vehicle Path Planning (VRP) problem is described as a cargo center that needs to deliver goods to multiple customer points by vehicles [22]. These goods points are in different locations, and the demand of each customer point is different and less than or equal to the vehicle's load capacity. The vehicle needs to transport goods within its own load capacity, not exceeding the vehicle's load capacity. Each customer point can only be distributed by a car. After distributing a number of customer points to return to the distribution center to form a loop, in the event of satisfying all the constraints, find a distribution route so that all the vehicles travel the longest distance. In order to meet all the constraints, it is important to find a distribution route that ensures that all vehicles travel

the shortest distance, in order to reduce transportation costs and improve distribution efficiency. Figure 1 shows the schematic diagram of the vehicle path problem, which consists of 1 cargo center and 14 customer points, all customer points are distributed by a total of 3 vehicles.

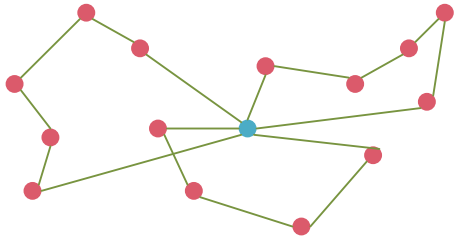


Figure 1. VRP diagram

2.2.2 Modeling the vehicle path problem

1) Problem

Let the distribution center be numbered 0, each demand point be numbered 1, 2, 3 .....N, the distribution center owns K distribution vehicles, and the load capacity of each car be Q. The departure points of the distribution vehicles are all at the same distribution center, delivering to N demand points, respectively. After each vehicle completes its task, it has to return to the departure point again to plan a satisfactory transportation route where the demand of demand point i is denoted as $qi(i=1,2,3\cdots\cdots N)$, the distance between two points of demand point i and demand point j is denoted as $Cij(i,\ j=1,2,3.....N)$, and the distance from the distribution center to the demand point is denoted as $C_{0j}(j=1,2,3.....N)$.

2) Objective

Minimum mileage of the traveled route aggregated by all the vehicles dispatched with the least total cost of distribution.

3) Decision variables:

$$y_k=\begin{cases}1 \text{ Demand point i is delivered by vehicle k} & i=1,2,3,\ldots\ldots N;k=1,2,3\ldots\ldots K\\ 0 \text{ Or else}\end{cases} \quad (1)$$

$$x_{ijk}=\begin{cases}1 \text{ Vehicle k travels from demand point i to demand point j} & i\neq j;i,j=1,2,3,.....N\\ 0 \text{ Or else}\end{cases} \quad (2)$$

4) Models:

$$\text{MinZ}=\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{K}c_{ij}x_{ijk} \quad (3)$$

s.t.

$$\sum_{i=1}^{N}q_i y_{ki}\le Q \quad (k=1,2,3.....K) \quad (4)$$

$$\sum_{k=1}^{K} y_{ki} = 1 \quad (i = 1,2,3,.....N; \quad k = 1,2,3.....K) \tag{5}$$

$$\sum_{i=1}^{N} x_{ijk} = y_{ki} \quad (j = 1,2,3,.....N; \quad k = 1,2,3.....K) \tag{6}$$

$$\sum_{j=1}^{N} x_{ijk} = y_{ki} \quad (i = 1,2,3,.....N; \quad k = 1,2,3.....K) \tag{7}$$

$$x_{ijk} \in \{0,1\} (i, j = 1,2,3,....N; \quad k = 1,2,3.....K) \tag{8}$$

$$y_{ki} \in \{0,1\} \quad (i = 1,2,3,.....N; \quad k = 1,2,3.....K) \tag{9}$$

Equations (1) and (2) represent the constraints on the decision variables. Equation (3) denotes that the objective function of vehicle path optimization is the minimum mileage transported by the vehicle, and Equation (4) denotes that the amount of goods loaded by each vehicle at the time of distribution is less than the upper limit of the vehicle's carrying capacity. Equation (5) indicates that each demand point cannot be served by more than one vehicle, i.e., there is only one vehicle for distribution service. Equation (6) indicates that if a demand point j is served by vehicle k for distribution service, then vehicle k must arrive at demand point j from demand point i. Equation (7) indicates that if the goods required at demand point i are fulfilled by vehicle k, then vehicle k completes the delivery service at demand point i and then completes the service experience for demand point j.

2.2.3 Common Algorithms for Optimizing Vehicle Path Problems

Adopting reasonable and scientific methods to optimize logistics and distribution path problems is an important research topic in logistics and distribution. At present, under the development of computer science, experts from various disciplines have proposed different optimal algorithms based on different types of constraint models and problems through a large number of theoretical research and experimental analyses, and scholars or experts have proposed different optimal algorithms for different situations, which are widely applied in the fields of logistics and transportation systems, distribution systems, and so on. It is commonly used in the fields of logistics transportation systems, distribution systems, and so on. The process of calculating logistics and distribution path optimization problems is complicated, and the selection of different algorithms may have an impact on the accuracy of the calculation. Figure 2 illustrates the classification of methods related to VPR problem-solving.

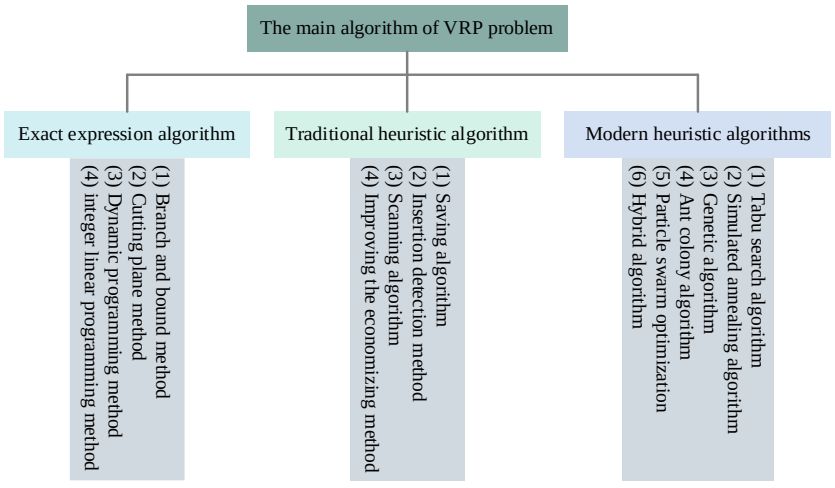


Figure 2. Classification of methods related to VRP problem-solving

3 Example study of the current status of logistics and distribution

3.1 Company profile

Since 1998, Company A has been a provider of daily necessities, which includes sourcing, packaging, express transportation, warehousing, full trucking, and supply chain as a comprehensive service-oriented enterprise. The company mainly operates its stores, and its transportation modes cover road, rail, and air. The company has always insisted on improving the quality of its customer services, strengthening the continuous innovation of its management and operation modes, providing high-quality and personalized professional services for multiple industries, and continuously expanding its service scope, which has made it a leading enterprise in the industry. The Company has continued to improve its service quality, strengthened self-innovation, utilized its self-developed information system to realize an online cargo tracking query function, opened special road steamship lines, and entered the express delivery business in order to accelerate the expansion of its development areas and make quality products its strategic goal. As of 2022, Company A has nearly 20,000 business outlets, covering 32 provincial administrative regions in China, and its transit center covers an area of over 1.5 million square meters. With the purpose of serving customers wholeheartedly, the company continues to develop and innovate, forming unique competitive advantages and is committed to providing professional and comprehensive cargo supply and transportation services. Company A has business all over the country, of which the service in H city began in 2008, with procurement, logistics, transportation, packaging, freight, bonded, road transport, and steamshipment as the main products, and the company has always been advocating the provision of professional services in H city, and has 67 business networks in H city so far, and has been providing professional services. So far, H city has 67 business outlets and transport vehicles nearly 100, the operation center covers an area of 20,000 square meters, H city has superior traffic conditions, the cargo transport business is larger, the number of customers served is increasing, always adhere to the purpose of providing high-quality service to customers, safe and efficient transportation to provide customers with reliable and convenient service, trusted by the majority of customers.

3.2 Status of Company A’s distribution business

3.2.1 Distribution business operation flow

To ensure that customers receive the required products on time and in quality and complete the company’s business transactions, the distribution business operation process of Company A is being implemented. Generally, the company’s customers must upload the product order form one day in advance, and the company’s back-office system will synchronously upload the order information to the distribution center’s warehouse management platform after receiving the order information. The company makes the goods according to the order information and leaves the distribution center at 6:00 a.m. On the following day, the distribution service will start. In addition, the company will combine the customer order quantity and warehouse inventory for product quantitative verification, confirm the product purchase information, and send it to the purchasing specialist in the main production area of high-quality products. The purchasing specialist can purchase the goods according to the product-related data information. Company A has an independent logistics and distribution center and distribution fleet because of the many customer groups and industries involved. Figure 3 is the flow chart of Company A’s distribution operations. Through the on-site inspection of Company A, the distribution operation flow of the company is roughly divided into eight steps: purchasing, inspecting and warehousing, sorting and processing, organizing and checking, packing and trucking, vehicle distribution, customer acceptance, and vehicle return.

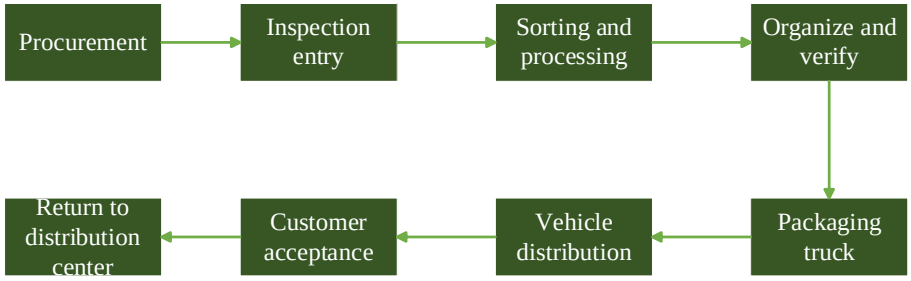


Figure 3. Distribution operation flowchart

3.2.2 Distribution Business Analysis

With the continuous development of the company's business, involving the expansion of the scope of services, Company A, in order to compete in the market more advantageous, so constantly improve the logistics and distribution of each link, the formation of "production, supply and marketing integration" mode. Among them, the enterprise's procurement, transportation, warehousing, and other links have been focused on improving, but they have also achieved certain results. However, there are more problems and certain complexities in the vehicle distribution link, and it isn't easy to formulate and implement the distribution path program, which also indicates that there is obvious room for improvement in this link.

Figure 4 shows the proportion of logistics costs in Company A, where DC, WC, TC, AC, and OC represent distribution costs, warehousing costs, transportation costs, management costs, and other costs, respectively. According to the research data, the distribution cost of Company A accounts for 42% of the total logistics cost, which is higher than other logistics costs. With the continuous expansion of the company's business scale, the logistics cost will continue to rise. The proportion of distribution cost will be even higher, which not only restricts the profitability of Company A's revenue but also impedes the capital investment in other areas, thus affecting Company A's overall development. Reducing the distribution cost is crucial for lowering the total logistics cost of Company A.

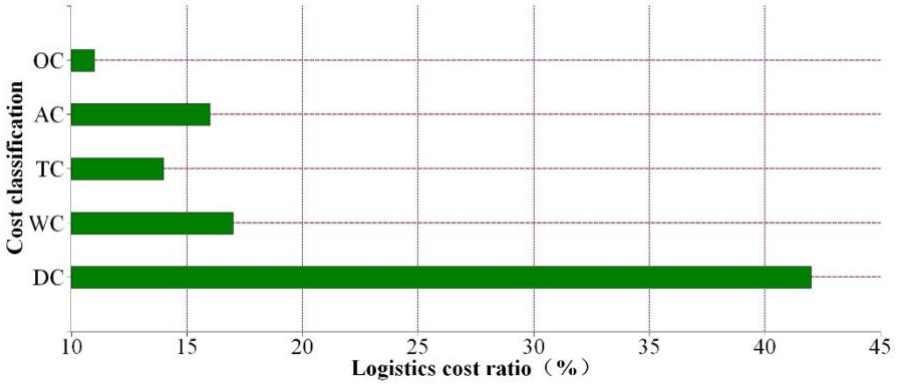


Figure 4. The logistics cost ratio of A Company

In addition, Company A currently has different distribution time requirements for each customer point. In actual distribution, according to the customer points assigned by the distribution administrator, distribution is selected only on the basis of the proximity of the distribution path, which determines the distribution time and the distance of distribution. Sometimes it is also the case that the most urgent customer's situation is distributed first, and the distribution route lacks planning and is relatively

chaotic, making it impossible to effectively control the distribution cost. Table 1 shows the average annual customer complaint survey statistics for Company A. The relevant data show that the time punctuality of delivery and product freshness accounted for 90.1%, and the rest of the problems, such as after-sales service and the service attitude of the delivery personnel, also accounted for 7.92% and 1.98%, respectively. The delivery process is influenced by the punctuality of delivery time and the freshness of delivered products. Company A's sustainability depends on improving the accuracy of delivery time and freshness of products to meet customers' needs.

Table 1. Customer Annual Average Complaint Survey Statistics

Month	Delivery time punctuality issue/order	After-sales service issue/order	Freshness issues with delivery products/order	Delivery personnel service attitude issue/order
1	8	1	7	1
2	4	2	5	0
3	7	1	7	0
4	5	4	9	0
5	9	2	10	0
6	7	1	13	1
7	11	0	14	0
8	8	1	11	2
9	10	1	8	0
10	5	1	5	0
11	7	0	5	0
12	4	2	3	0
Total	85	16	97	4
Proportion	42.08%	7.92%	48.02%	1.98%

3.3 Analysis of logistics and distribution problems in Company A

As a comprehensive service enterprise that has been established for more than twenty years, Company A has rich experience in logistics and distribution and can complete the distribution tasks only by relying on the drivers' own experience. However, the lack of reasonable distribution path planning, even if successful completion of the distribution task was achieved, caused part of the cost loss, which affected the profitability of Company A's income. Therefore, in order to "reduce costs and increase efficiency", re-planning distribution routes and formulating a scientific and effective distribution program is of vital importance for Company A. After conducting field research, we found that Company A's main problems are irrational distribution route planning. Low customer satisfaction is caused by inadequate distribution; low average vehicle load factor results in a high total cost of product distribution; and Company A does not have a suitable alternative distribution program when customer demand surges.

1) Irrational distribution route planning: Company A's distribution route relies on the driver's own experience in distribution without comprehensively analyzing the actual needs of customers, which easily leads to an increase in the distribution distance, generating additional vehicle transportation costs, increasing the company's cost expenditures, and creating a burden on the company.

2) Inopportune delivery and low customer satisfaction. Customers require drivers to deliver on time as much as possible while performing delivery tasks and providing high-quality delivery services. If the driver, based on their own experience, only according to the shortest delivery distance and the location of the customer point of proximity to select the delivery order, then the distribution of the distance and the actual delivery of the arrival time will increase. The actual delivery of the arrival time is often not to the customer's satisfaction with the time range.

3) The average load factor of vehicles is low, and the total cost of product distribution is too high. Company A has nearly 100 distribution vehicles, but the average load factor of cars is only 62.01%. Company A, in order to ensure the freshness of the products and give customers a good shopping experience, uses refrigerated trucks to distribute fresh products in a low-temperature environment, which generates additional refrigeration costs for Company A compared to companies that do not use refrigerated trucks.

4) Company A lacks a suitable alternative distribution program when there is a surge in customer demand. Usually, customers do not easily change their buying habits, and the number and types of products they buy do not change much. Despite holiday promotions, customer demand will fluctuate. Therefore, Company A needs to be equipped with a distribution plan to respond quickly to customer demand when there is a surge in demand.

4 Logistics and distribution optimization strategy based on mixed integer linear programming

To solve logistics and distribution problems, improve customer satisfaction, and increase the company's revenue and profit, this section will optimize the company's logistics and distribution strategy. Focusing on the use of mixed integer linear programming methods to optimize the vehicle path of the company's logistics and distribution so as to improve the punctuality of logistics and distribution, reduce certain distribution costs, and achieve the expected effect of "improving quality and efficiency".

4.1 Mixed integer linear programming

4.1.1 Variable substitution

The variable $e_{ij} = v_{ij}^2$ is introduced as a new decision variable to reconstruct and optimize the vehicle path problem model proposed earlier [23]. The model Eq. (1) and Eq. (2) can be rewritten as:

$$\min_{x_i, e_i} \frac{1}{1000\eta} \sum_{(i,j)\in A} d_{ij} x_{ij} (0.5 C_d \rho A e_{ij} + Mg \sin\theta_{ij} + C_r Mg \cos\theta_{ij}) \tag{10}$$

s.t.

$$\sum_{(i,j)\in A} x_{ij} \frac{d_{ij}}{\sqrt{e_{ij}}} \leq T \tag{11}$$

$$l_{ij}^2 \leq e_{ij} \leq u_{ij}^2, \forall (i,j) \in A \tag{12}$$

4.1.2 Linear optimization of logistics and distribution

1) Objective function

For the multiplication of integer and continuous variables in the objective function equation, define the new continuous variable z_{ij} and the following conditions:

$$z_{ij},, \overline{z}_{ij} x_{ij}, \quad \forall (i, j) \in A \tag{13}$$

$$z_{ij},, 0.5C_d \rho A e_{ij} + Mg \sin\theta_{ij} + C_r Mg \cos\theta_{ij}, \forall (i, j) \in A \tag{14}$$

$$z_{ij}..0.5C_d \rho A e_{ij} + Mg \sin\theta_{ij} + C_r Mg \cos\theta_{ij} - \overline{z}_{ij}(1 - x_{ij}), \forall (i, j) \in A z_{ij}..0, \forall (i, j) \in A \tag{15}$$

Where $\overline{z}_{ij}$ is an upper bound on $0.5C_d \rho A e_{ij} + Mg \sin\theta_{ij} + C, Mg \cos\theta_{ij}$ and can be set to $0.5C_d \rho A u_{ij}^2 + Mg \sin\theta_{ij} + C_r Mg \cos\theta_{ij}$.

The objective function equation can be converted to:

$$\min_{x_\alpha, e_\alpha, z_\alpha} \frac{1}{1000\eta} \sum_{(i,j)\in A} d_{ij} z_{ij} \tag{16}$$

2) Time constraint

The nonlinear term $\frac{1}{\sqrt{e_{\bar{y}}}}$ in Eq. causes some difficulties in the solution. This paper adopts the segmented linear function [24] and uses the tangent method to select the segmented straight line. In order to facilitate the illustration, take the 3-segmented straight line as an example. The segmented linear function formula is as follows:

$$f_{ij}(e_{ij}) = \begin{cases} \lambda_{ij1} e_{ij} + \gamma_{ij1}, & e_{ij} \in [l_{ij}^2, E_{ij1}) \\ \lambda_{ij2} e_{ij} + \gamma_{ij2}, & e_{ij} \in [E_{ij1}, E_{ij2}) \\ \lambda_{ij3} e_{ij} + \gamma_{ij3}, & e_{ij} \in (E_{ij2}, u_{ij}^2] \end{cases} \quad \forall (i, j) \in A \tag{17}$$

Where $\lambda_{ij1}, \lambda_{ij2}, \lambda_{ij3}$ is the slope of each segment of the straight line, $\gamma_{ij1}, \gamma_{ij2}, \gamma_{ij3}$ is the intercept of each segment of the straight line, E_{ij1} is the demarcation point of the straight line between segment 1 and segment 2, and E_{ij2} is the demarcation point of the straight line between segment 2 and segment 3.

Next, the three segments of the straight line are merged using a multiple-choice model [25]. Defining the new continuous variable $q_{ij1}, q_{ij2}, q_{ij3}$ and the 0-1 variable $\sigma_{ij1}, \sigma_{ij2}, \sigma_{ij3}$, the segmented linear function can be written as:

$$f_{ij}(e_{ij}) = \sum_{k=1}^{3} (\lambda_{ijk} q_{ijk} + \gamma_{ijk} \sigma_{ijk}), \quad \forall (i, j) \in A(20) \tag{18}$$

s.t.

$$e_{ij} = \sum_{k=1}^{3} q_{ijk}, \quad \forall (i, j) \in A \tag{19}$$

$$l_{ij}^2 \sigma_{ij1}, q_{ij1}, E_{ij1}\sigma_{ij1}, \quad \forall (i, j) \in A \tag{20}$$

$$E_{ij1}\sigma_{ij2}, q_{ij2}, E_{ij2}\sigma_{ij2}, \quad \forall (i, j) \in A \tag{21}$$

$$E_{ij2}\sigma_{ij3}, q_{ij3}, u_{ij}^2 \sigma_{ij3}, \quad \forall (i, j) \in A \tag{22}$$

$$\sigma_{ij1} + \sigma_{ij2} + \sigma_{ij3}, 1, \quad \forall (i, j) \in A \tag{23}$$

$$\sigma_{ij1}, \sigma_{ij2}, \sigma_{i\beta} \in \{0,1\}, \quad \forall (i, j) \in A \tag{24}$$

Extend the segmented linear functional equation for a 3-segment line to the general case of a n-segment line. Let the transverse coordinates of the dividing point between the straight lines be $\{E_{ij1}, E_{ij2}, \cdots, E_{ij(n-1)}\}$ and satisfy $l_{ij}^2 < E_{ij1} < E_{ij2} < \cdots < E_{ij(n-1)} < u_{ij}^2$ let $E_{ij0} = l_{ij}^2, E_{ijn} = u_{ij}^2$, then the general expression is:

$$f_{ij}\left(e_{ij}\right) = \sum_{i=1}^{n} \left(\lambda_{ijk} q_{ijk} + \gamma_{ijk} \sigma_{ijk}\right), \quad \forall (i, j) \in A \tag{25}$$

s.t.

$$e_{ij} = \sum_{k=1}^{n} q_{ijk}, \forall (i, j) \in A \tag{26}$$

$$E_{ij(k-1)}\sigma_{ijk}, q_{ijk}, E_{ijk}\sigma_{ijk}, \tag{27}$$

$$\forall (i, j) \in A, k \in \{1, 2, \cdots, n\} \tag{28}$$

$$\sum_{k-1}^{n} \sigma_{ijk}, 1, \quad \forall (i, j) \in A \tag{29}$$

$$\sigma_{ijk} \in \{0,1\}, \quad \forall (i, j) \in A \tag{30}$$

Therefore, the constraint equation can be written as:

$$\sum_{(i,j)\in A} d_{ij} x_{ij} \left[\sum_{k=1}^{n} (\lambda_{ijk} q_{ijk} + \gamma_{ijk} \sigma_{ijk})\right], T \tag{31}$$

It is worth noting that constraint Eq. (26)-Eq. (31) is a relaxation of constraint Eq. (11) since the value of the segmented linear function $f_{ij}(e_{ij})$ is less than or equal to the value of $\frac{1}{\sqrt{e_{ij}}}$ due to the use of tangent lines for the approximation of the nonlinear term $\frac{1}{\sqrt{e_{ij}}}$. The accuracy can be improved in practical applications by increasing the number of segments.

The multiplication of integer and continuous variables in Eq. (31) can be done in a similar way as before. Assuming that M_{ij} is an upper bound for $f_{ij}(e_{ij})$ and defining a new continuous variable y_{ij}, Eq. (31) can be replaced by the following condition:

$$\sum_{(i,j)\in A} d_{ij} y_{ij},, T \tag{32}$$

$$y_{ij},, M_{ij} x_{ij}, \quad \forall (i,j) \in A \tag{33}$$

$$y_{ij},, \sum_{k=1}^{n} (\lambda_{ijk} q_{ijk} + \gamma_{ijk}), \quad \forall (i,j) \in A \tag{34}$$

$$y_{ij}.. \sum_{k=1}^{n} (\lambda_{ijk} q_{ijk} + \gamma_{ijk}) - M_{ij}(1 - x_{ij}), \quad \forall (i,j) \in A \tag{35}$$

$$y_{ij}..0, \quad \forall (i,j) \in A \tag{36}$$

Ultimately, the original vehicle path problem models Eqs. (1) and (2) are linearized into a new mixed-integer linearized model: the objective function Eq. (16) and a set of constraints that can be solved quickly by many planning solvers.

4.2 Application examples and results analysis

4.2.1 Example data

In this paper, we select the actual logistics order data of one of the outlets of Company A in City H in August 2022. The number of orders to be delivered by the distribution center of the outlet on that day is 24. We plan to provide a reasonable transportation solution for each vehicle in the distribution center.

It is known that the distribution center has five vehicles, all of which are small freight cars. The maximum load of the car is 2 tons (2000 kg), the fixed start-up cost of each vehicle is 100 yuan, the price per kilometer of each car is 2.5 yuan, the vehicle loading and unloading of goods are 3 minutes, other time consumption is ignored for the time being. The maximum penalty cost of each vehicle is 50 yuan. On a given day, 25 customers are required to transport goods from a distribution center. The cars in this distribution center start delivering at 5 a.m. Table 2 shows the relative horizontal and vertical axes, the demand for goods, and the acceptable and required time windows for each customer location in a city [26].

Table 2. The coordinate position of the customer and the quantity of goods

Customer number	X-coordinate (km)	Y-coordinate (km)	Demand for goods (kg)	Acceptable time window (min)	Required time window (min)
1	147	142	48	(300,420)	(340,380)
2	43	158	143	(540,660)	(580,620)
3	112	181	149	(980,1100)	(1020,1060)
4	87	164	77	(640,760)	(300,340)
5	89	242	216	(900,1020)	(940,980)
6	63	217	279	(700,820)	(740,780)
7	209	258	163	(292,412)	(332,372)
8	162	226	132	(1064,1184)	(1104,1144)
9	209	211	231	(708,828)	(748,788)
10	173	182	267	(901,1021)	(941,981)
11	261	247	91	(474,594)	(514,554)
12	249	174	308	(1080,1200)	(1120,1160)
13	192	187	264	(641,761)	(681,721)
14	187	213	315	(763,883)	(803,843)
15	269	142	191	(1103,1223)	(1143,1183)
16	138	108	102	(377,497)	(417,457)
17	227	70	328	(1109,1229)	(1149,1189)
18	219	112	283	(998,1118)	(1038,1078)
19	68	83	69	(1146,1266)	(1186,1226)
20	94	124	329	(1024,1144)	(1064,1104)
21	107	60	301	(437,557)	(477,517)
22	48	99	337	(589,709)	(629,669)
23	145	74	291	(828,948)	(868,908)
24	43	129	122	(383,503)	(423,463)

4.2.2 Analysis of Results

Mixed integer linear programming aims at the goal of improving the quality of the solution, so in order to prove that it can achieve a good distribution path solution when solving the vehicle path optimization problem with a time window, two comparative studies were conducted. The first study uses an unoptimized model of the vehicle path problem, while the second study uses an optimized model of the vehicle path problem using mixed integer linear programming. The results of the two studies are compared to verify the effectiveness of mixed integer linear programming for the vehicle path problem in logistics and distribution.

The results of the study are made highly comparable and convincing by using the same set of actual data in both studies for research verification. The study is conducted with all the platforms and hardware devices being consistent, and the specific operating environment is as follows: the CPU is 3.40GHz, the running memory is 8GB, the operating system is Windows 10, and the research software is Matlab2019a.

1) Research results of vehicle path problem modeling

Obtaining the specific transport route of each vehicle in this distribution center can be achieved by initializing the parameters. Figure 5 shows the unoptimized vehicle paths, and the red, blue, gray, green, and magenta line schematics in the figure indicate the vehicle distribution routes from No. 1 to No. 5, respectively. Each vehicle begins at the distribution center, completes the distribution task at its assigned customer point in accordance with the route diagram, and then returns to the distribution center. The loading of each vehicle, the actual path length traveled, the total transportation cost, and the specific direction of the route are shown in Table 3. It can be seen that car 1, traveling the red route, and car 2, traveling the blue route, have fewer customers assigned to them. In contrast, the three vehicles traveling the gray, green, and magenta routes have more customers assigned to them, and the transportation routes are more complex. Penalty costs exist for all five cars, with car 1 having a lesser cost of 30.47 yuan and car 3 having the highest penalty cost of 73.68 yuan, indicating that the vehicle has the highest penalty cost due to the delivery process of delayed or early delivery resulted in high penalty costs, and the service quality of the delivery was average.

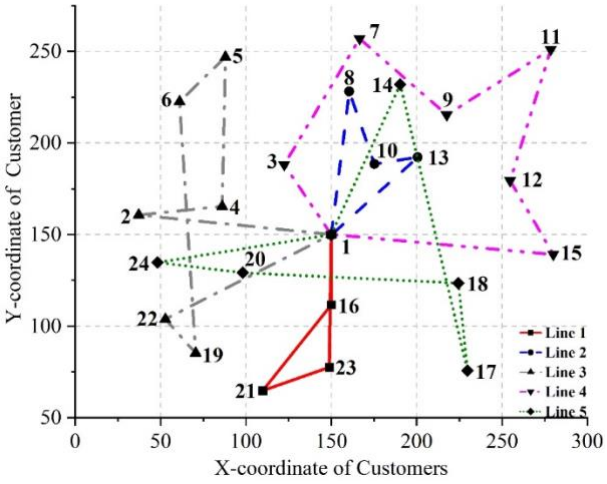


Figure 5. Does not optimize the path diagram of the vehicle

Table 3. Vehicle-specific driving data

Vehicle number	Actual load(kg)	Actual distance (km)	Time penalty cost (CNY)	Total transport cost (CNY)	Transport route
1	864	208.37	30.47	561.28	1-16-21-23-1
2	703	214.58	35.29	563.86	1-8-10-13-1
3	1049	549.91	73.68	1447.58	1-2-4-5-6-19-22-1
4	1238	517.82	52.16	1359.04	1-3-7-9-12-15-1
5	1247	584.43	44.89	1531.77	1-14-17-18-24-24-1

2) Optimized vehicle path model research results

In order to optimize the path of logistics and distribution vehicles, reduce the penalty fees and other logistics costs due to delays and other reasons, and improve the quality of logistics and distribution services, the original vehicle path problem model is optimized using mixed integer linear programming. The specific transport route diagram of each vehicle in the distribution center obtained by executing the optimized model is shown in Figure 6. The red, blue, gray, green, and magenta lines in the figure represent vehicles 1 to 5, respectively. Each vehicle

starts from the distribution center, completes the distribution task of its classified customer points according to the route map in turn, and then returns to the distribution center. The figure shows that the vehicle traveling the red path is assigned a relatively small amount of customers, only 2 customers. The other 4 vehicles are assigned a more uniform number of customers. The complexity of the transport route and route distance traveled compared to the original vehicle path problem model has been greatly improved. The specific vehicle loading of each vehicle, the actual path length traveled, the total transportation cost, and the specific direction of each route are shown in Table 4. After path optimization, the 1st vehicle does not incur penalty costs, indicating that all customers delivered by this vehicle are able to be delivered on time with better service quality. The other 4 vehicles have penalty costs, but the costs are greatly reduced compared to the original vehicle path model. Among them, the penalty cost of the third vehicle decreases by 66.40%. The vehicle path can effectively reduce vehicle penalty cost and improve distribution quality by using mixed integer linear programming, as demonstrated.

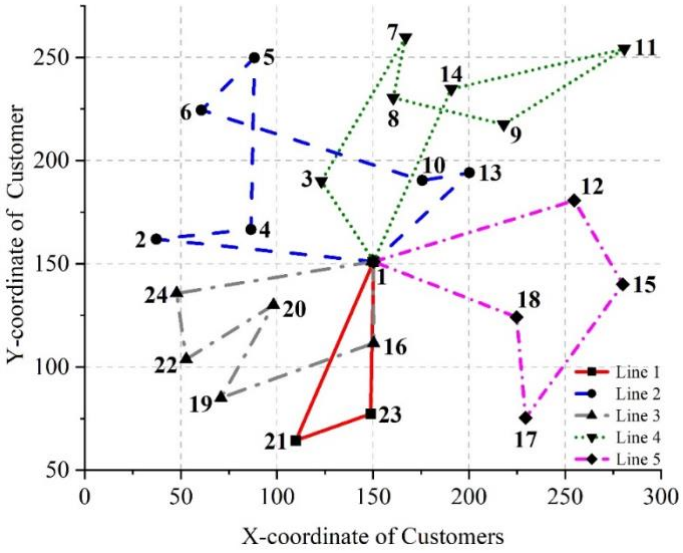


Figure 6. The vehicle path after optimization

Table 4. Vehicle-specific driving data

Vehicle number	Actual load(kg)	Actual distance (km)	Time penalty cost (CNY)	Total transport cost (CNY)	Transport route
1	671	207.43	0	503.28	1-21-23-1
2	1148	483.26	37.81	1007.56	1-2-4-5-6-10-13-1
3	1209	351.72	24.76	922.13	1-16-19-20-22-24-1
4	1386	455.03	21.43	1015.77	1-3-7-8-9-11-14-1
5	1126	301.86	10.87	908.34	1-12-15-17-18-1

5 Conclusion

This paper takes the logistics distribution problem of Company A as an example. It draws the following conclusions by comparing the results of logistics distribution before and after utilizing mixed integer linear programming:

1) In the analysis of the current situation of logistics and distribution represented by Company A, the distribution cost of Company A accounts for 42% of the total cost of logistics, which

seriously limits the company's profitability income and affects the overall development of Company A. The statistical results of the survey of the average annual customer complaints of Company A show that the problem of distribution time punctuality and the problem of freshness of the distributed products account for 42.08% and 48.02%, respectively. To summarize the results, the current logistics and distribution services are generally characterized by irrational distribution route planning, inopportune distribution, low customer satisfaction, and high total cost of product distribution.

2) The number of customers assigned by the optimized vehicle is more evenly distributed, and the complexity of the transportation route and the route driving distance is also greatly improved compared with the original vehicle path planning model. After path optimization, the penalty cost of the 1st vehicle is 0, and the penalty cost of the other 4 cars is also reduced significantly, in which the 3rd vehicle with the highest penalty cost before optimization has a 66.40% decrease in penalty cost after model optimization. Using mixed integer linear programming to optimize the vehicle path can effectively improve the rationality of vehicle path planning, properly solve the existing problems in logistics and distribution services, and improve the quality and efficiency of logistics and distribution services.

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