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THE APPLICATION OF METHODS OF ROBUST M-ESTIMATION IN ESTABLISHING SHIP POSITION IN MARINE TRAFFIC SURVEILLANCE SYSTEMS BASED ON RADAR OBSERVATIONS

ABSTRACT

In the last years considerable emphasis has been placed on safety at sea. There is the maritime security and surveillance system whose main aim is to execute tasks in the interests of maritime safety and to react in case of emergency. They are monitored by networks of radar stations.

On such areas we obtain a lot of navigation data which could be used to improve ship's parameters (position), using know in geodesy modern M-estimation methods. Simultaneous acquisition of navigational information from many independent radar stations will render it possible to obtain a more accurate ship position in marine traffic surveillance systems in relation to the calculated position. A position expected in an adjustment calculus is received from a watch officer. It is burdened with a fallacy of navigation systems and the quality of marking ship's route on a map. In the case of navigational-parameter measurements used for depicting ship position, one can obtain incorrect results due to a disturbance in the measurement process. In extreme cases, such erroneous data could significantly differ from the anticipated results. Deviating observations could significantly influence the values of measurement results. In order to eliminate the determination of erroneous measurements, one could use resistant estimation methods with suitably selected attenuation functions. The accuracy of a determined position will not be better than the capabilities of the device used. Adjustment gives the possibility of eliminating or minimizing human errors as well as the errors in the indications of navigational devices.

This paper presents the latest robust estimation methods using Danish attenuation function for adjustment of navigational observation, using radar observation.

Keywords:
M-estimation, radar, traffic systems.

INTRODUCTION

Determination of ship positions applying terrestric navigation is much more error biased if compared to fixing thereof with high accuracy satellite navigation. The weakness of GPS system is its users. A damaged or malfunctioning receiver may affect safety of navigation. Also GPS signal can easy be interfered with some other equipment. Transmitting at frequencies of 1450 up to 1600 MHz with some watts power is fairly enough to disorientate GPS receivers within a range of several meters. Thus, navigation at sea may not be based on GPS system only. Today the users usually rely on GPS system and forget to keep it continuously under control and check is the GPS fixed position the right one [2, 3].

It is necessary to remember that in spite of lower levels of accuracy, terrestric navigation is still indispensable. Recent development of technology enabled to raise the accuracy. It's the radar navigation which is at this moment of specific importance. At waterways with heavy traffic there operate the radar systems which are a part of traffic surveillance systems. They keep monitoring vessels traffic in the system and provide information on their positions in real time. Arrangement of radar stations along the coast can be used to obtain at the same time information about any vessel. For such structures, coordinates of the points which are applied to equalize the ship position, are usually used as the parameters.

Obtaining navigational information from many independent radar stations simultaneously, enables to acquire more accurate data referring to the vessels' positions in the ships traffic surveillance systems. On carrying out navigation, the errors connected with observed positions depend on human factor and errors in vessel navigational equipment readings. It is the same with radar observation. The research works in field automation of radar navigation was created by [5, 6] too.

The traffic surveillance systems comprise always several radio-location coastal stations. They acquire at the same time navigational parameters (bearing or distance), necessary to determine vessels' positions on waterways. In case there are measured the navigational parameters required to define the vessel location, in a result of any survey process disturbance we may obtain results biased with serious errors, and in some extreme situations the errors may greatly differ from the anticipated ones. The errors of such nature will be named gross errors. Whenever any gross error occurs, it is advised to rectify measurement and correct the results. Making resurveys for a vessel sailing at water area is impossible. Within every time interval the ship covers a certain distance of the route and does not ‘come back’ to the last survey point. Thus the errors may cause improper navigation and in consequence endanger safety of navigation.

For the surveyed navigational measurements there may be applied the commonly known in geodesy M-estimation methods, allowing to determine the accurate vessel’s position, received from the watch officer, basing on the obtained navigational radar observations. The fixed position accuracy will not be higher than the used equipment survey capability. Equalization enables to eliminate or minimize human errors and also ship navigational equipment indications errors.

The advanced M-estimation methods can greatly correct such errors and limit their influence on plotting the route on plot charts. In the classic equalization, when the least squares method is applied, the survey result is a random variable, characterized with the same standard deviation. Each result is treated the same way. The outlaying observations may affect meaningly the survey results values and therefore the applied method is not robust to gross errors. To enable elimination of determining false radar echoes, there may be applied the methods of robust estimation of adequately selected attenuation functions. One of them is Danish attenuation function, which was applied for calculations and presented in the Paper.

METHODS OF ROBUST ESTIMATION

For navigation purpose there are often applied the geometric measurement structures, which may be defined in (X, Y) system, where the parameter to be measured is bearing. In Fig. 1 an exemplary survey grid is presented; it comprises coastal radar stations used for making surveys. The traffic surveillance systems cover some stations located along the whole coastal line; in various geometric configurations they can be used for creation of radar survey network [8].

For such a structure can perform measurements of a vessel at Z_j position from coastal radar station. Assuming different time intervals, there can be measured one of navigational parameters, as for example the bearing. In practice there occurs that survey results are biased with serious errors, often referred to as gross errors. To restrict an influence of such errors, there can be used the methods of robust estimation, of suitably selected attenuation functions. For purpose of the presented studies the Danish attenuation function was applied.

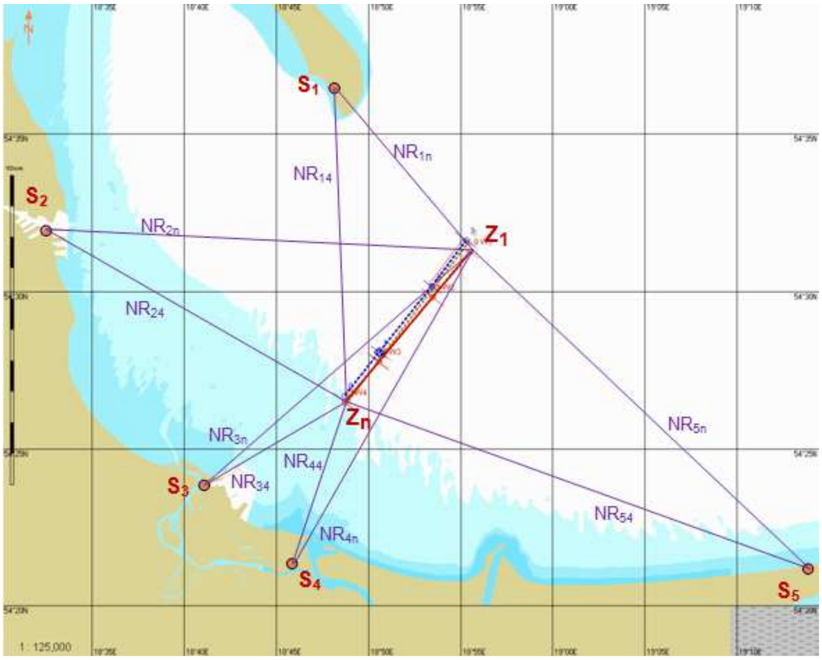

Fig. 1. Measurement structure [own study]

The Danish attenuation function is characterized with such properties, that beyond the admissible range $\Delta\bar{v}$, it decreases ex-potentially. The following is a form of the attenuation function [8]:

$$t(\bar{v}_n) = \begin{cases} 1 & \text{for} \quad \bar{v}_n \in \langle -k; k \rangle \\ \exp\left\{-l(|\bar{v}_n| - k)^g\right\} & \text{for} \quad |\bar{v}_n| > k \end{cases} \tag{1}$$

In that case the equivalent weights values are settled according to the formula [8]:

$$\hat{p}_n = t(\bar{v}_n) p_n = \begin{cases} p_n & \text{for} \quad \bar{v}_n \in \langle -k; k \rangle \\ \exp\left\{-l(|\bar{v}_n| - k)^g\right\} p_n & \text{for} \quad |\bar{v}_n| > k \end{cases} \tag{2}$$

Assuming that the measured navigational parameter (bearing), in effect of — for example — radar echo misidentification, is gross error biased, then to such observation there will be assigned the so-called equivalent weight $\hat{\mathbf{P}}_\mathbf{n}$, which is a result of attenuation of the original weight $\mathbf{P}$, resulting from the assumed mean error of survey. Proceeding of the attenuation process follows according to the dependence [8]:

$$\widehat{\mathbf{P}}_{\mathbf{n}} = \mathbf{T}\left(\overline{\mathbf{V}}_{\mathbf{n}}\right)\mathbf{P}_{\mathbf{n}} \tag{3}$$

where

$\mathbf{T}\left(\overline{\mathbf{V}}_{\mathbf{n}}\right)$ is the function of attenuation of the following basic properties:

for $v_i \in \Delta v_i, v_j \in \Delta v_j: \quad t(v_i) = t(v_j)$.

The intervals $\Delta v_i = \left\langle -k\sigma_{v_i}; k\sigma_{v_i} \right\rangle$ are intervals admissible for corrections $v_i,\ i = 1,\dots,n$, settled at the assumed confidence level γ. Assuming that v_i are random variables characterized with normal distributions, it may be put down that:

$$\gamma = P(-k\sigma_{v_i} < v_i < k\sigma_{v_i}) = P(-k < \bar{v}_i < k) = \int_{-k}^{+k} \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{\bar{v}_i^2}{2}\right] d\bar{v}_i \tag{4}$$

where:

$\sigma_{v_i} = \sqrt{[\mathbf{C}_V]_{ii}}$ — standard deviation of the *i*-th correction;

$\bar{v}_i = \dfrac{v_i}{\sigma_{v_i}}$ — standardized corrections of common admissible interval $\Delta\bar{v}_i = <-k; k>$.

For example: for $\gamma = 0.95$ coefficient $k = 2$.

The equivalent weights matrix $\widehat{\mathbf{P}} = \mathbf{T}\left(\overline{\mathbf{V}}\right)\mathbf{P}$ is dependent on the standardized corrections vector $\overline{\mathbf{V}}$. The equalizing problem with application of the method robust to gross errors may take the following form [8]:

$$\left.\begin{array}{lr}
\mathbf{V} = \mathbf{A}\hat{\mathbf{d}}_{\mathrm{X}} + \mathbf{L} & \text{functional model} \\
\mathbf{C}_{\mathbf{X}} = \sigma_0^2 \mathbf{Q}_{\mathbf{X}} = \sigma_0^2 \mathbf{P}^{-1} & \text{original statistic model} \\
\widehat{\mathbf{C}}_{\mathbf{X}} = \sigma_0^2 \widehat{\mathbf{Q}}_{\mathbf{X}} = \sigma_0^2 \widehat{\mathbf{P}}^{-1} & \text{equivalent statistic model} \\
\widehat{\mathbf{P}} = \mathbf{T}\left(\overline{\mathbf{V}}\right)\mathbf{P} & \text{equivalent weights} \\
\min\limits_{\mathbf{d}_{\mathrm{X}}} \left\{\xi\left(\hat{\mathbf{d}}_{\mathrm{X}}\right) = \mathbf{V}^{\mathrm{T}}\widehat{\mathbf{P}}\mathbf{V} = \mathbf{V}^{\mathrm{T}}\mathbf{T}\left(\overline{\mathbf{V}}\right)\mathbf{P}\mathbf{V}\right\} = & \\
\qquad = \hat{\mathbf{V}}^{\mathrm{T}}\mathbf{T}\left(\hat{\overline{\mathbf{V}}}\right)\mathbf{P}\hat{\mathbf{V}} & \text{equalization criterion}
\end{array}\right\} \tag{5}$$

where:

$\widehat{\mathbf{P}} = \mathbf{T}\left(\overline{\mathbf{V}}\right)\mathbf{P}$ — the equivalent weights matrix;

$\widehat{\mathbf{C}}_{\mathbf{X}}$ — equivalent co-variance matrix;

$\widehat{\mathbf{Q}}_{\mathbf{X}}$ — equivalent co-factors matrix;

$\mathbf{T}(\overline{\mathbf{V}})$ — is the following diagonal attenuation matrix.

For such accepted assumptions, solution of the equalizing problem, it means determination of such $\hat{\mathbf{d}}_{\mathrm{X}}$, that $\xi(\hat{\mathbf{d}}_{\mathrm{X}}) = \mathbf{V}^{\mathrm{T}}\widehat{\mathbf{P}}\mathbf{V} = \min$ is of an iterative character. In the first steps of the iterative process solving the equalizing problem, it is assumed that $l = 0{,}01 \div 0{,}1,\ g = 2$. Improperly selected parameters cause unnecessary increase of a number of steps in the iterative process, solving the robust problem of equalizing. To solve the problem there can be accepted the proposed in [7, 8], algorithm, where the first stage of the calculation process is equalizing applying the classic least squares method. Then the observational equations system can be presented as follows:

$$NR_{ij}^{Z} = arctg \left. \frac{Y_{S_i} - Y_{Z_j}}{X_{S_i} - X_{Z_j}} \right|_{\substack{i=1,\ldots,n \\ j=1,\ldots,m}} \tag{6}$$

where:

(X_{S_i}, Y_{S_i}) — coordinates of the radar stations locations;

(X_{Z_i}, Y_{Z_i}) — ship position coordinates.

In the equalization process we assume that the observed position, transferred by a watch officer to the traffic surveillance system operator, is the anticipated position of the coordinates:

$$\mathrm{Z}_{\mathrm{j}}^{0} = \begin{bmatrix} \mathrm{X}_{\mathrm{j}}^{0} \\ \mathrm{Y}_{\mathrm{j}}^{0} \end{bmatrix} \tag{7}$$

for the survey structure accepted for these considerations, the equalizing problem's functional model takes the form as follows:

$$\mathrm{v}_{\mathrm{ij}} = \left. \frac{\partial \mathrm{NR}_{\mathrm{ij}}}{\partial \mathrm{X}_{\mathrm{Z_j}}} \hat{\mathrm{d}}_{\mathrm{X}_{\mathrm{Z_j}}} + \frac{\partial \mathrm{NR}_{\mathrm{ij}}}{\partial \mathrm{Y}_{\mathrm{Z_j}}} \hat{\mathrm{d}}_{\mathrm{Y}_{\mathrm{Z_j}}} + \mathrm{NR}_{\mathrm{ij}}^{0} - \mathrm{NR}_{\mathrm{ij}}^{\mathrm{obs}} \right|_{\substack{\mathrm{i}=1,\ldots,\mathrm{n} \\ \mathrm{j}=1,\ldots,\mathrm{m}}} \tag{8}$$

where:

v_{ij} — correction of the measured bearing from the *i*-th radar station ($i = 1, \ldots, 5$);

NR_{ij}^0 — radar bearing value for $Z_j^0\left(X_j^0, Y_j^0\right)$;

Z_j — *j*-th position of the ship ($j = 1, \ldots, 4$).

Assuming that:

$$\mathbf{V} = \begin{bmatrix} v_{11} \\ v_{21} \\ v_{31} \\ v_{41} \\ v_{51} \end{bmatrix} \text{ — corrections vector} \qquad \mathbf{A} = \begin{bmatrix} \frac{\partial NR_{11}}{\partial X_{Z_1}} & \frac{\partial NR_{11}}{\partial Y_{Z_1}} \\ \frac{\partial NR_{21}}{\partial X_{Z_1}} & \frac{\partial NR_{21}}{\partial Y_{Z_1}} \\ \frac{\partial NR_{31}}{\partial X_{Z_1}} & \frac{\partial NR_{31}}{\partial Y_{Z_1}} \\ \frac{\partial NR_{41}}{\partial X_{Z_1}} & \frac{\partial NR_{41}}{\partial Y_{Z_1}} \\ \frac{\partial NR_{51}}{\partial X_{Z_1}} & \frac{\partial NR_{51}}{\partial Y_{Z_1}} \end{bmatrix} \text{ — matrix of coefficients}$$

at unknowns

$$\hat{\mathbf{d}}_X = \begin{bmatrix} \hat{d}_{X_{Z_1}} \\ \hat{d}_{Y_{Z_1}} \end{bmatrix} \text{ — the searched vector of increments to the anticipated coordinates}$$

$$\mathbf{L} = \begin{bmatrix} NR_{11}^0 - NR_{11}^{obs} \\ NR_{21}^0 - NR_{21}^{obs} \\ NR_{31}^0 - NR_{31}^{obs} \\ NR_{41}^0 - NR_{41}^{obs} \\ NR_{51}^0 - NR_{51}^{obs} \end{bmatrix} \text{ — free terms matrix}$$

Then the matrix system of corrections equation takes the form as follows:

$$\mathbf{V} = \mathbf{A} \cdot \hat{\mathbf{d}}_X + \mathbf{L} \tag{9}$$

With the indeterminate method applied, a solution of this equation system is:

$$\hat{\mathbf{d}}_X = -\left(\mathbf{A}^{\mathbf{T}}\mathbf{PA}\right)^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{PL} \tag{10}$$

where:

$$\mathbf{P} = \begin{bmatrix} \frac{1}{m_{ij}^2} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{m_{ij}^2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{m_{ij}^2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{m_{ij}^2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{m_{ij}^2} \end{bmatrix}$$ — is the carried out observations weights matrix

(i = 1, …, 5), (j = 1, ..., 4)

m_{ij}^2 — mean error of the *i*-th observation to the *j* — the position of the ship.

Thus the estimators of the equalized coordinates of a vessel at sea are as follows:

$$\hat{\mathbf{Z}}_{\mathbf{j}} = \mathbf{Z}_{\mathbf{j}}^{\mathbf{0}} + \hat{\mathbf{d}}_{XY} = \begin{bmatrix} X_j^0 \\ Y_j^0 \end{bmatrix} + \begin{bmatrix} \hat{d}_{X_{Z_j}} \\ \hat{d}_{Y_{Z_j}} \end{bmatrix} = \begin{bmatrix} \hat{X}_j \\ \hat{Y}_j \end{bmatrix} \tag{11}$$

where:

$\hat{\mathrm{Z}}_{\mathrm{j}}\left(\hat{\mathrm{X}}_{\mathrm{j}}, \hat{\mathrm{Y}}_{\mathrm{j}}\right)$ — estimated position of a vessel at sea.

To determine the carried out observations quality it is necessary to define a statistic model of the equalizing problem:

$$\hat{\mathbf{C}}_{\hat{\mathbf{X}}} = m_0^2\left(\mathbf{A}^{\mathrm{T}}\mathbf{PA}\right)^{-1} = \begin{bmatrix} m_{\hat{\mathrm{X}}_{\mathrm{Z_j}}}^2 & \hat{\mathrm{c}}\mathrm{ov}\left(\hat{X}_j, \hat{Y}_j\right) \\ \hat{\mathrm{c}}\mathrm{ov}\left(\hat{X}_j, \hat{Y}_j\right) & m_{\hat{\mathrm{Y}}_{\mathrm{Z_j}}}^2 \end{bmatrix} \tag{12}$$

where:

$m_0^2 = \dfrac{V^T PV}{n-r}$ — variance coefficient estimator;

n — number of the carried out observations;
r — number of the unknowns.

Then the mean error of the observed position can be determined applying the dependence:

$$M_{po} = \sqrt{m^2_{\hat{X}_{Z_j}} + m^2_{\hat{Y}_{Z_j}}} \tag{13}$$

To settle which of the standardized corrections may represent the gross errors (not covered by $\Delta\bar{v}$, the corrections vector's covariance matrix for $m_0 = 1$ has to be determined

$$\hat{\mathbf{C}}_{\hat{\mathbf{V}}(m_o=1)} = \mathbf{P}^{-1} - \mathbf{A}\left(\mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{T}} \tag{14}$$

followed by classification to verify is it for each:
$i := \bar{v}_i \in \Delta\bar{v}$.

If all $\bar{v}_i \in \Delta\bar{v}$, then we omit further calculations, as the acquired values of the standardized corrections are comprised within the interval acceptable thereto and the newly determined attenuation matrix will not cause decreasing the weights matrix values and after that the corrections values (within the accepted calculations accuracy limits). Otherwise, if any correction $\bar{v}_i \notin \Delta v$, then the next iteration steps are carried out. Assuming that $j = 0$ and then $\mathbf{V}^{(j)} = \mathbf{V}$, $\mathbf{P}^{(j)} = \mathbf{P}$, $\mathbf{C}^{(j)}_{\mathbf{V}(m_o=1)} = \hat{\mathbf{C}}^{(j)}_{\hat{\mathbf{V}}(m_o=1)}$ and also the parameters controlling in the Danish attenuation function $l = 0.01$, $g = 2$ it is computed as follows.
Reminding that:

$$t(\bar{v}_n) = \begin{cases} 1 & \text{for} \quad \bar{v}_n \in \langle -k; k \rangle \\ \exp\left\{-l\left(|\bar{v}_n| - k\right)^g\right\} & \text{for} \quad |\bar{v}_n| > k \end{cases} \tag{15}$$

we calculate the attenuation function values:

$$\bar{v}_i^{(j)} \in \Delta\bar{v} \;\rightarrow\; t\left(\bar{v}_i^{(j)}\right) = 1$$
$$\bar{v}_i^{(j)} \notin \Delta\bar{v} \;\rightarrow\; t\left(\bar{v}_i^{(j)}\right) = \exp\left\{-l\left(|\bar{v}_n| - k\right)^g\right\}$$

and the attenuation matrix

$$\mathbf{T}\left(\overline{\mathbf{V}}^{(j)}\right)=\begin{bmatrix} t\left(\overline{v}_i^{(j)}\right) & & & \\ & \ddots & & \\ & & \ddots & \\ & & & \ddots \\ & & & & t\left(\overline{v}_n^{(j)}\right) \end{bmatrix}_{\text{dla } i=1,\ldots,n} \tag{16}$$

Next we carry out iteration, decrease j by 1, it means: $j := j + 1$ and carry out calculations of the weights matrix, increments and corrections matrix:

$$\left.\begin{aligned} &\mathbf{P}^{j} = \mathbf{T}\left(\overline{\mathbf{V}}^{(j-1)}\right)\mathbf{P}^{(j-1)} \\ &\mathbf{d}_X^{(j)} = -\left(\mathbf{A}^{T}\mathbf{P}^{(j)}\mathbf{A}\right)^{-1}\mathbf{A}^{T}\mathbf{P}^{(j)}\mathbf{L} \\ &\mathbf{V}^{(j)} = \mathbf{A}\cdot\mathbf{d}_X^{(j)} + \mathbf{L} \end{aligned}\right\} \tag{17}$$

For the assumed accuracy of the calculations we check differences between the corrections vectors elements:

$$\mathbf{V}^{(j)} \quad \text{i} \quad \mathbf{V}^{(j-1)}$$

In case the differences are bigger than the assumed ones, we compute the corrections vector's covariance matrix for $m_0 = 1$.

$$\mathbf{C}_{\mathbf{V}\,(m_o=1)}^{(j)} = \left(\mathbf{P}^{(j)}\right)^{-1} - \mathbf{A}\left(\mathbf{A}^{T}\mathbf{P}^{(j)}\mathbf{A}\right)^{-1}\mathbf{A}^{T}$$

$$m_{v_i}^{(j)} = \sqrt{\left[\mathbf{C}_{\mathbf{V}\,(m_o=1)}^{(j)}\right]_{ii}} \qquad \overline{v}_i^{(j)} = \frac{v_i^{j}}{m_{v_i}^{(j)}}$$

Then we calculate a value of the attenuation function and the attenuation matrix. After increasing j by 1, it means: $j := j+1$ we start the next step of iteration. Iteration is finished with such equalization, wherein the obtained standardized corrections values are included within a range admissible for them, and the resulting new attenuation matrix is not causing decreasing the weights matrix values, and next the correction values (within the accepted calculations accuracy limits). The final weights matrix is an equivalent matrix, whereas a solution obtained basing thereon is the final solution. In the equivalent weights matrix, the weights corresponding to the gross errors biased observations are no longer the original weights (with the values resulting from the mean errors of survey), but weights of values decreased or sometimes equal to zero.

TEST

Within the selected survey network, from five coastal radar stations there were carried out observations, focused on a vessel, traveling at the Gulf of Gdańsk area. The navigational observations consisted in measuring the bearings. Due to the extensive survey sequence length, the article displays measurements taken for four positions of the vessel. Values of the bearings are presented in Table 1. For purpose of the presented research, one measurement was gross error biased; such situation can happen in practice in case the radar operator takes bearing improperly or in case an object is misidentified. For the first two positions Z_1,Z_2 the gross error appears with the bearing taken from the coastal station in Gdynia, and for the next two vessel's positions Z_3, Z_4 the errors comes out with the bearing taken from the station in Gdańsk.

Table 1. The surveyed coordinates and bearings of the ship at the Gulf of Gdańsk area [own study]

Survey point	The coastal radar stations					Z_j^0
	Hel Lighthouse	Gdynia Harbour Master	Gdańsk North Port Harbour Master	Górki Zachodnie Radar Tower	Krynica Morska Lighthouse	
Z_1	140,2°	93,3°	47,6°	29,5°	294,1°	φ = 54° 31.279' N λ = 18° 55.539' E
Z_2	156,1°	100,5°	49,3°	27,3°	288,8°	φ = 54° 29.829' N λ = 18° 53.472' E
Z_3	172,8°	112,5°	53,3°	21,8°	282,7°	φ = 54° 27.769' N λ = 18° 50.539' E
Z_4	180,2°	121°	58,1°	15,1°	278,4°	φ = 54° 26.485' N λ = 18° 48.712' E

Coordinates of the coastal radiolocation stations wherefrom the bearings were taken are presented in Table 2.

Table 2. Coordinates of the coastal radar stations [own study]

Lp.	Coastal station	Rectangular coordinates (X,Y)
1	Hel Lighthouse	X = 6052469,34 Y = 358694,38
2	Gdynia Harbour Master	X = 6045676,69 Y = 341307,40
3	Gdańsk North Port Harbour Master	X = 6030435,11 Y = 350472,86
4	Górki Zachodnie Radar Tower	X = 6027021,91 Y = 355714,18
5	Krynica Morska Lighthouse	X = 6027542,89 Y = 399407,33

Due to the restrictions connected with the Paper size, the Authors present results of the calculations carried out for Z1 position. Results of all equalizations are displayed in Fig. 2 and Table 3.

To simplify the calculation process it was decided that the further calculations are to be performed in the rectangular coordinates system instead of the geographic coordinates system. The calculations were carried out taking into consideration each attitude of the ship at a moment of taking the bearing. To reach clarity of the Paper only the first calculations were presented.

The determined by the watch officer coordinates of a ship at sea are assumed to be the anticipated coordinates of the ship. For such the observational system it is assumed the corrections equations system (6) for which, after substitution of the above data, the following elements of the matrix corrections equations system (9) are obtained.

In effect of equalizing the observations carried out in the first step with the least squares method the following results are obtained. Matrix of coefficients with the unknowns:

$$\mathbf{A} = \begin{bmatrix} -0.00311 & -0.00395 \\ -0.00233 & -0.00021 \\ -0.00217 & 0.00185 \\ -0.00154 & 0.00254 \\ 0.00139 & 0.00066 \end{bmatrix}$$

Matrix of free terms:

$$\mathbf{L}=\begin{bmatrix}140.2-141.83\\70.3-95.08\\47.6-49.43\\26.5-31.31\\293.3-295.37\end{bmatrix}=\begin{bmatrix}1.685\\24.78\\1.84\\1.81\\1.47\end{bmatrix}_{[\circ]}$$

Assuming that the mean error of the observation is $m_{ij}=0.5^\circ$ for $i=1,...,5$ also assuming that the carried out observations are independent, the observations weights matrix takes the form as follows:

$$\mathbf{P}=\begin{bmatrix}4&0&0&0&0\\0&4&0&0&0\\0&0&4&0&0\\0&0&0&4&0\\0&0&0&0&4\end{bmatrix}_{[\mathrm{m}]}$$

The vector of increments to the anticipated coordinates is:

$$\hat{\mathbf{d}}_{\mathrm{X}}=-\left(\mathbf{A}^{\mathrm{T}}\mathbf{PA}\right)^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{PL}=\begin{bmatrix}\hat{d}_{X_{Z_1}}\\\hat{d}_{Y_{Z_1}}\end{bmatrix}=\begin{bmatrix}2933.31\\-548.69\end{bmatrix}_{[m]}$$

Thus the corrections vector takes the value:

$$\mathbf{V}=\begin{bmatrix}v_{\mathbf{1}}\\v_2\\v_3\\v_4\\v_5\end{bmatrix}=\begin{bmatrix}-5.32\\18.07\\-5.53\\-4.11\\5.18\end{bmatrix}$$

Therefore the following is the estimator of the observed vessel's position at sea, applying the observations made from the coastal stations as well:

$$\hat{\mathrm{Z}}_1=\begin{bmatrix}\hat{X}_1\\\hat{Y}_1\end{bmatrix}=\begin{bmatrix}X_1^0\\Y_1^0\end{bmatrix}+\begin{bmatrix}\hat{d}_{X_{Z_1}}\\\hat{d}_{Y_{Z_1}}\end{bmatrix}=\begin{bmatrix}6043505.62\\365741.39\end{bmatrix}+\begin{bmatrix}2933.31\\-548.69\end{bmatrix}=\begin{bmatrix}6046438.93\\365192.70\end{bmatrix}$$

Whereas the carried out observations estimators are respectively:

$$\begin{bmatrix} \hat{NR}_{11} \\ \hat{NR}_{21} \\ \hat{NR}_{31} \\ \hat{NR}_{41} \\ \hat{NR}_{51} \end{bmatrix} = \begin{bmatrix} 134.88 \\ 88.37 \\ 42.07 \\ 25.39 \\ 199.08 \end{bmatrix}_{[\circ]}$$

The mean error of the estimated vessel coordinates is accordingly:

$$m_{\hat{X}_{Z_1}} = \sqrt{3764343.04} = 1940.19\,[\mathrm{m}]$$

$$m_{\hat{Y}_{Z_1}} = \sqrt{3489458.84} = 1868.01\,[\mathrm{m}]$$

Finally the mean error of the position is:

$$M_{Z_1} = \sqrt{m^2_{\hat{X}_{Z_1}} + m^2_{\hat{Y}_{Z_1}}} = 2693.29\,[\mathrm{m}]$$

Basing on the obtained results it has to be defined which of the standardized corrections may represent gross errors. Assuming for calculations $\gamma = 0.95$, wherefrom $k = 2$. The admissible interval $\Delta\overline{V}$ is of the form as follows: $\Delta\overline{V} \in \langle -k;k \rangle = \langle -2;2 \rangle$. To settle which of the standardized corrections estimators may represent gross errors (not covered by $\Delta\overline{V}$), there is determined the corrections vector covariance matrix for $m_0 = 1$:

$$\hat{\mathbf{C}}_{\hat{\mathbf{V}}(m_o=1)} = \mathbf{P}^{-1} - \mathbf{A}\left(\mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{T}} = \begin{bmatrix} 0.04501 & -0.06372 & 0.00747 & 0.04501 & 0.05553 \\ -0.06372 & 0.19253 & -0.06073 & -0.04758 & 0.03243 \\ 0.00747 & -0.06073 & 0.14416 & -0.10473 & 0.02337 \\ 0.04501 & -0.04758 & -0.10473 & 0.13945 & 0.01261 \\ 0.05553 & 0.03243 & 0.02337 & 0.01261 & 0.22885 \end{bmatrix}$$

and then there is carried out the following classification:

$$\bar{\hat{v}}_1 = \frac{(\hat{v}_1)_{(m)}}{(\sqrt{0.04501})_{(m)}} = -25.056 \quad \notin \Delta\bar{v}$$

$$\bar{\hat{v}}_2 = \frac{(\hat{v}_2)_{(m)}}{(\sqrt{0.19253})_{(m)}} = 41.175 \quad \notin \Delta\bar{v}$$

$$\bar{\hat{v}}_3 = \frac{(\hat{v}_3)_{(m)}}{(\sqrt{0.14416})_{(m)}} = -14.576 \quad \notin \Delta\bar{v}$$

$$\bar{\hat{v}}_4 = \frac{(\hat{v}_4)_{(m)}}{(\sqrt{0.13945})_{(m)}} = -10.997 \quad \notin \Delta\bar{v}$$

$$\bar{\hat{v}}_5 = \frac{(\hat{v}_5)_{(m)}}{(\sqrt{0.22885})_{(m)}} = 10.833 \quad \notin \Delta\bar{v}$$

The obtained results prove that none of the standardized corrections' estimators lies within the admissible interval range. Wherever the methods common in geodesy are applied, any measurements taken with such errors should be rejected and the measurements resurveyed. Anyhow it is difficult for a port approaching ship to turn back and enable taking measurements once more when she is at the same positions. However, being not influenced by such errors, we can equalize again the measurement results, making observations results robust to gross errors with application of the Danish function of attenuation.

Assuming $\mathbf{V}^{(0)} = \hat{\mathbf{V}}$, $\mathbf{P}^{(0)} = \mathbf{P}$ and $l = 0.02$, $g = 2$ (in the Danish attenuation function the controlling parameters), let's calculate:

— the attenuation function value (where $\Delta\bar{v} \in \langle -k;k \rangle = \langle -2;2 \rangle$)

$$\bar{v}_1^{(0)} = \bar{\hat{v}}_1 \quad \notin \Delta\bar{v} \quad \rightarrow \quad t\left(\bar{v}_1^{(0)}\right) = \exp\{-l(25.056 - k)^g\} = 2.414 \cdot 10^{-5}$$

$$\bar{v}_2^{(0)} = \bar{\hat{v}}_2 \quad \notin \Delta\bar{v} \quad \rightarrow \quad t\left(\bar{v}_2^{(0)}\right) = \exp\{-l(41.175 - k)^g\} = 4.68 \cdot 10^{-14}$$

$$\bar{v}_3^{(0)} = \bar{\hat{v}}_3 \quad \notin \Delta\bar{v} \quad \rightarrow \quad t\left(\bar{v}_3^{(0)}\right) = \exp\{-l(14.576 - k)^g\} = 0.042$$

$$\bar{v}_4^{(0)} = \bar{\hat{v}}_4 \quad \notin \Delta\bar{v} \quad \rightarrow \quad t\left(\bar{v}_4^{(0)}\right) = \exp\{-l(10.997 - k)^g\} = 0.198$$

$$\bar{v}_5^{(0)} = \bar{\hat{v}}_5 \quad \notin \Delta\bar{v} \quad \rightarrow \quad t\left(\bar{v}_5^{(0)}\right) = \exp\{-l(10.833 - k)^g\} = 0.21$$

— matrix of attenuation

$$\mathbf{T}\left(\overline{\mathbf{V}}^{(0)}\right)=\begin{bmatrix} t\left(\bar{v}_1^{(0)}\right) & & & & \\ & t\left(\bar{v}_2^{(0)}\right) & & & \\ & & t\left(\bar{v}_3^{(0)}\right) & & \\ & & & t\left(\bar{v}_4^{(0)}\right) & \\ & & & & t\left(\bar{v}_5^{(0)}\right) \end{bmatrix}=\begin{bmatrix} 2.414\cdot 10^{-5} & & & & \\ & 4.68\cdot 10^{-14} & & & \\ & & 0.042 & & \\ & & & 0.198 & \\ & & & & 0.21 \end{bmatrix}$$

The, applying the iteration method, we perform computations; at the step 1 there are calculated:

— weights matrix

$$\widehat{\mathbf{P}}^{(1)}=\mathbf{T}\left(\overline{\mathbf{V}}^{(0)}\right)\mathbf{P}^{(0)}=\begin{bmatrix} 9.65461\cdot 10^{-5} & & & & \\ & 1.87183\cdot 10^{-13} & & & \\ & & 0.16915 & & \\ & & & 0.79252 & \\ & & & & 0.84014 \end{bmatrix}$$

— coordinates' increments

$$\hat{\mathbf{d}}_X^{(1)}=-\left(\mathbf{A}^T\widehat{\mathbf{P}}^{(1)}\mathbf{A}\right)^{-1}\mathbf{A}^T\widehat{\mathbf{P}}^{(1)}\mathbf{L}=\begin{bmatrix} -474.15 \\ -1067.99 \end{bmatrix}_{(m)}$$

— corrections vector

$$\hat{\mathbf{V}}^{(1)}=\mathbf{A}\cdot\hat{\mathbf{d}}_X^{(1)}+\mathbf{L}=\begin{bmatrix} 7.32 \\ 26.1 \\ 0.88 \\ -0.17 \\ 0.11 \end{bmatrix}_{(m)}$$

— diagonal's elements of matrix $\mathbf{C}_{\mathbf{V}(m_0=1)}^{(1)}$

$$\mathbf{C}_{\mathbf{V}(m_0=1)}^{(1)} = \begin{bmatrix} 1.035\cdot 10^{4} & & & & \\ & 1.87183\cdot 10^{-13} & & & \\ & & 0.16915 & & \\ & & & 0.79252 & \\ & & & & 0.84014 \end{bmatrix}$$

— value of the attenuation function

$$\begin{aligned} \overline{v}_1^{(1)} &= 0.072 \quad \in \Delta\overline{v} \;\rightarrow\; t\left(\overline{v}_1^{(1)}\right) = 1 \\ \overline{v}_2^{(1)} &= 1.129 \quad \in \Delta\overline{v} \;\rightarrow\; t\left(\overline{v}_2^{(1)}\right) = 1 \\ \overline{v}_3^{(1)} &= 0.403 \quad \in \Delta\overline{v} \;\rightarrow\; t\left(\overline{v}_3^{(1)}\right) = 1 \\ \overline{v}_4^{(1)} &= -0.401 \quad \in \Delta\overline{v} \;\rightarrow\; t\left(\overline{v}_4^{(1)}\right) = 1 \\ \overline{v}_5^{(1)} &= 0.221 \quad \in \Delta\overline{v} \;\rightarrow\; t\left(\overline{v}_5^{(1)}\right) = 1 \end{aligned}$$

— attenuation matrix

$$\mathbf{T}\left(\overline{\mathbf{V}}^{(1)}\right) = \begin{bmatrix} t\left(\overline{v}_1^{(0)}\right) & & & & \\ & t\left(\overline{v}_2^{(0)}\right) & & & \\ & & t\left(\overline{v}_3^{(0)}\right) & & \\ & & & t\left(\overline{v}_4^{(0)}\right) & \\ & & & & t\left(\overline{v}_5^{(0)}\right) \end{bmatrix} = \begin{bmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & 1 \end{bmatrix}$$

As all the standardized corrections remain within the interval of the admissible therefore, thus the attenuation matrix becomes the unit matrix and the first step is the one to finish the process of iteration of the equalizing problem, robust to gross errors. Finally the following solution is obtained at the end:

$$\hat{\mathbf{d}} = \hat{\mathbf{d}}_X^{(1)} = -\left(\mathbf{A}^T\widehat{\mathbf{P}}^{(1)}\mathbf{A}\right)^{-1}\mathbf{A}^T\widehat{\mathbf{P}}^{(1)}\mathbf{L} = \begin{bmatrix} -474.15 \\ -1067.99 \end{bmatrix}_{(m)}$$

$$\hat{\mathbf{V}} = \hat{\mathbf{V}}^{(1)} = \mathbf{A} \cdot \hat{\mathbf{d}}_X^{(1)} + \mathbf{L} = \begin{bmatrix} 7.32 \\ 26.1 \\ 0.88 \\ -0.17 \\ 0.11 \end{bmatrix}_{(m)}$$

$$\hat{\mathbf{C}}_{\hat{X}} = m_0^2 \left(\mathbf{A}^T \hat{\mathbf{P}} \mathbf{A} \right)^{-1} = \begin{bmatrix} 1.994 \cdot 10^4 & 9.929 \cdot 10^3 \\ 9.929 \cdot 10^3 & 1.419 \cdot 10^4 \end{bmatrix}$$

$$m_{\hat{X}_{z_1}} = 141.21[m] \qquad m_{\hat{Y}_{z_1}} = 119.12[m]$$

$$M_{po} = \sqrt{m^2_{\hat{X}_{z_1}} + m^2_{\hat{Y}_{z_1}}} = 184.74[m]$$

Having the final calculations done, one may perform visualization of the anticipated values and the estimated ones. Table 3 presents the increments and the estimated coordinates of the ship.

Table 3. Positions of the ship, increments and estimated coordinates [own study]

Survey point	Reckoned coordinates (X, Y)	Increments dx	Estimated coordinates (X, Y)
Z_1	X = 6043505.62 Y = 365741.39	X = 526.45 Y = –66.37	X = 6044032.07 Y = 365675.01
Z_2	X = 6040883.49 Y = 363431.40	X = 540.56 Y = 106.00	X = 6041424.06 Y = 363537.40
Z_3	X = 6037160.10 Y = 360149.01	X = 404.81 Y = 285.48	X = 6037564.92 Y = 360434.49
Z_4	X = 6034840.44 Y = 358101.69	X = 15.59 Y = 190.20	X = 6034856.03 Y = 358291.89

The graphical interpretation of Table 3 data is presented in Fig. 2.

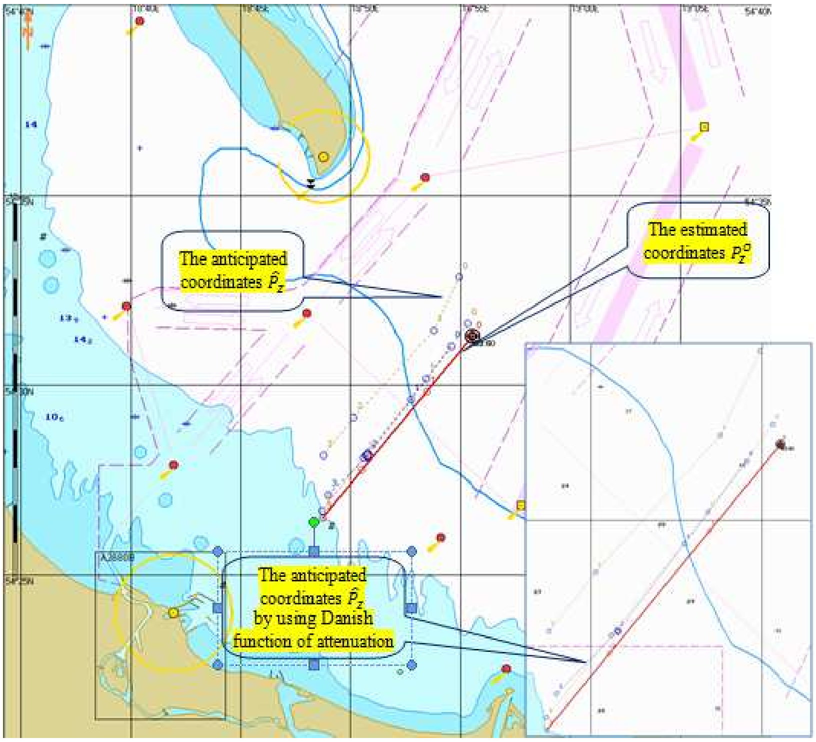

Fig. 2. The anticipated coordinates at P_Z^0 (in red) and the estimated coordinates $\hat{P}_Z^0$ (in yellow) determined upon carrying out the research [own study]

CONCLUSIONS

The main source of information on the movement of vessels which are transferred to the Vessel Traffic System in the areas of water is a network of coastal radar stations. Acquisition of navigational information simultaneously from many independent radar stations enables obtaining much more accurate positions of vessels provided by the traffic surveillance systems if compared to positions observed and worked out by vessels navigators [1].

At Polish sea areas at present the information on ships traffic is acquired from only one coastal station, chosen depending on which one is necessary. Acquisition of information from several radar stations concerning movable objects may contribute to new quality of navigation safety assessment, owing to more accurate and full information on any objects travelling within a range of radar stations operation.

Determination of the accurate position would surely increase safety of sailing within coastal areas owing to transfer to ships bridges the information essential for safe navigation.

Upon carrying out observations with a use of radar we may misread surveyed bearings. The outlaying observations may considerably affect the survey results' values. To reduce an influence of gross errors on ship's position determination, there may be applied the robust estimation methods with adequately selected attenuation functions, affecting accuracy of determining positions. The presented in the Paper M-estimation method applied with a use of the Danish attenuation function fulfils this criterion.

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STRESZCZENIE

W ostatnich latach znaczny nacisk kładzie się na bezpieczeństwo na morzu. Jest to zasadniczy cel tworzenia morskich systemów nadzoru i bezpieczeństwa, które powinny zareagować w razie pojawienia się zagrożenia. Ten nadzór jest realizowany poprzez sieć stacji radiolokacyjnych. Na takich obszarach otrzymujemy dużo danych nawigacyjnych, które mogłyby zostać użyte dla poprawienia parametrów statku (pozycji) przy użyciu nowoczesnych metod M-estymacji znanych z geodezji.

Równoczesne pozyskiwanie informacji nawigacyjnej z wielu niezależnych stacji radiolokacyjnych stwarza możliwości otrzymania dokładniejszej pozycji statku niż odbierana dotychczas w morskich systemach nadzoru ruchu morskiego. Pozycja estymowana w rachunku wyrównawczym jest otrzymywana od oficera wachtowego. Jest ona obarczona błędami systemów nawigacyjnych i jakości zaznaczenia trasy statku na mapie. W wypadku pomiaru parametrów nawigacyjnych używanych dla przedstawiania pozycji statku można otrzymać niepoprawne wyniki z powodu zakłóceń procesu pomiarowego, a błędne obserwacje mogą znacząco wpłynąć na wyniki. Dla wyeliminowania złych pomiarów można by użyć odpornych metod estymacji z odpowiednio dobraną funkcją wzmocnienia. Dokładność wyznaczonej pozycji nie będzie większa niż możliwości użytych systemów pomiarowych. Estymacja daje możliwość eliminowania albo minimalizacji błędów ludzkich oraz błędów we wskazaniach urządzeń nawigacyjnych.

Artykuł przedstawia najnowszą metodę estymacji z wykorzystaniem tzw. duńskiej funkcji wzmocnienia dla dostosowania obserwacji nawigacyjnych opartych na obserwacjach radarowych.