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THE N -FUZZY METRIC SPACES AND MAPPINGS WITH APPLICATION

ABSTRACT. In the present paper, we introduce the notion of N -fuzzy metric spaces ($NFMS_s$), Pseudo N -fuzzy metric spaces and describe some of their properties. Also we prove a fixed point theorem using implicit relation in N -fuzzy metric spaces.

KEY WORDS: N -fuzzy metric space, pseudo- N -fuzzy metric space, fixed point, compatible maps, weak compatible maps, semicompatible maps, compatible maps, the property $(E.A.)$.

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1. Introduction

In 1975, Kramosil and Michalek [14] introduced the concept of fuzzy metric space. George and Veeramani [11] modified the concept of fuzzy metric space. In 1963, Gähler [9, 10] generalized usual notion of metric space called 2-metric space.

Using the notion of 2-metric space, S. Sharma [26] and S. Kumar [15] introduced fuzzy-2-metric spaces without knowing each other but Ha et al. in [12] shows that 2-metric need not be continuous function, further there is no easy relationship between results obtained in the two settings. In 1992, Bapure Dhage [7] in his Ph.D thesis introduced a new class of generalized metric space called D-metric space [7]. B. Singh and M. Chouhan [27] defined S -fuzzy metric space by using the concept of D-metric space [7]. However, Mustafa and Sims in [16] have pointed out that most of the results claimed by Dhage and others in D-metric spaces are invalid. To overcome these fundamental flaws, they introduced a new concept of generalized metric space called G-metric space [17]. Using the concept of G-metric space, G. Sun and K Yang [29] introduced the notion of Q-Fuzzy metric space, K.P.R. Rao et. al. [20] proved two fixed point theorems in symmetric Q(G)-metric space, Sedghi et. al. in [25] introduced D^* -metric space which

is a generalization of G -metric space and gave an example which is D^* metric space but not G -metric space. Using the concept of D^* -metric space, Sedghi and Shobe [23] defined M -fuzzy metric space. Very recently, Sedghi et. al [24] defined S -metric space which is a generalization of D^* -metric space and G -metric space and justified their work by various examples and definitions related to topology of S -metric space. The recent literatures of fixed point in fuzzy metric spaces is also given in [3, 4, 5, 6, 8, 19].

In present paper, after preliminaries in section 3, we define N -fuzzy metric space using S -metric space [24] which generalized $Q(orG)$ fuzzy metric space and M -fuzzy metric space. We also define open balls for topology, convergent sequence, Cauchy sequence and continuous functions and prove various interesting lemmas related to topology and convergent sequence in the NFM space. In Section 4, we define compatible maps, weak compatible maps, semi compatible maps and property $(E.A.)$ with counter examples in the NFM space. In Section 5, we define Pseudo- N -fuzzy metric space with counter example and recall definition implicit relation from [1]. Finally in Section 6, we extend a fixed point theorem of Irshad Aalam et. al [1] using implicit relation in the structure of NFM space with appropriate example.

2. Preliminaries

We recall the following definitions of fuzzy metric spaces and S -metric space.

Definition 1 ([21]). *A mapping $*$: $[0, 1] \times [0, 1] \rightarrow [0, 1]$ is called a continuous t -norm if $([0, 1], *)$ is an abelian topological monoid with unit 1 such that $a * b \leq c * d$ for $a \leq c, b \leq d$. Examples of t -norms are $a * b = \min\{a, b\}$ (minimum t -norm) $a * b = ab$ (product t -norm) and $a * b = \max\{a + b - 1, 0\}$ (Lukasiewicz t -norm).*

Definition 2 ([11]). *A 3-tuple $(X, M, *)$ is said to be a fuzzy metric space if X is an arbitrary non empty set, $*$ is a continuous t -norm, and M is a fuzzy set on $X^2 \times (0, \infty)$ satisfying following conditions for all $x, y \in X, s, t > 0$:*

$$M_1 : M(x, y, t) > 0$$

$$M_2 : M(x, y, t) = 1 \text{ if and only if } x = y$$

$$M_3 : M(x, y, t) = M(y, x, t)$$

$$M_4 : M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$$

$$M_5 : M(x, y, \cdot) : (0, \infty) \rightarrow (0, 1] \text{ is a continuous function.}$$

The pair $(M, *)$ (or only M) is called a fuzzy metric on X . Here $M(x, y, t)$ is considered as the degree of nearness of x and y with respect to t . The axiom M_1 is justified because a classical metric cannot take the value ∞ then

M cannot take the value 0. The axiom M_2 is equivalent to the following: $M(x, x, t) = 1$ for all $x \in X$ and $t > 0$ and $M(x, y, t) < 1$ for all $x \neq y$ and $t > 0$. The axiom M_2 gives the idea that when $x = y$ the degree of nearness of x and y is perfect or simply 1 and then $M(x, x, t) = 1$ for each $x \in X$ and for each $t > 0$. Finally, in M_5 we assume that the variable t behave nicely, that is assume that for fixed x and y , $t \rightarrow M(x, y, t)$ is a continuous function. The reader can refer to the examples and related definitions of fuzzy metric space in [11] and [14].

Remark 1. The above definition of George-Veeramani is a modified version of Kramosil-Michalek. This modification is necessary since the topology induced by the fuzzy metric in definition of Kramosil-Michalek is not Hausdorff.

Definition 3 ([15]). A map $*: [0, 1] \times [0, 1] \times [0, 1] \rightarrow [0, 1]$ is called a continuous t -norm if it satisfies the following conditions:

$$T_1 : *(a, 1, 1) = a, *(0, 0, 0) = 0$$

$$T_2 : *(a, b, c) = *(a, c, b) = *(b, c, a)$$

$$T_3 : *(a_1, b_1, c_1) \geq *(a_2, b_2, c_2) \text{ for } a_1 \geq a_2, b_1 \geq b_2, c_1 \geq c_2$$

examples of t -norm are (1) : $a * b * c = a.b.c$ and (2) : $a * b * c = \min\{a, b, c\}$

Definition 4 ([15, 26]). The triplet $(X, M, *)$ is a fuzzy 2-metric space if X is an arbitrary non empty set, $*$ is a continuous t -norm and M is a fuzzy set on $X^3 \times (0, \infty)$ satisfying following conditions for all $x, y, z, a \in X$, $r, s, t > 0$:

$$M_1 : M(x, y, a, t) > 0$$

$$M_2 : M(x, y, a, t) = 1 \text{ if and only if at least two of } x, y, z \text{ are equal.}$$

$$M_3 : M(x, y, a, t) = M(y, a, x, t) = M(a, y, x, t) \text{ (symmetry)}$$

$$M_4 : M(x, y, a, r + s + t) \geq M(x, y, z, r) * M(x, z, a, s) * M(z, y, a, t)$$

$$M_5 : M(x, y, a, \cdot) : (0, \infty) \rightarrow (0, 1] \text{ is left continuous}$$

$$M_6 : \lim_{t \rightarrow \infty} M(x, y, a, t) = 1.$$

For examples and related definitions of fuzzy 2-metric space the reader can refer [15] and [26].

Definition 5 ([27]). A 3-tuple $(X, S, *)$ is said to be an S -fuzzy metric space if X is an arbitrary non empty set, $*$ is a continuous t -norm and S is a fuzzy set on $X^3 \times (0, \infty)$ satisfying following conditions for all $x, y, z, w \in X$, $r, s, t > 0$:

$$(i) S(x, y, z, t) > 0$$

$$(ii) S(x, y, z, t) = 1 \text{ if and only if } x = y = z$$

$$(iii) S(x, y, z, t) = S(y, z, x, t) = S(z, y, x, t) \dots$$

$$(iv) S(x, y, z, r + s + t) \geq S(x, y, w, r) * S(x, w, z, s) * S(w, y, z, t)$$

$$(v) S(x, y, z, \cdot) : (0, \infty) \rightarrow (0, 1] \text{ is continuous.}$$

For more on S -fuzzy metric space, the reader may consult in [27].

Definition 6 ([22]). A 3-tuple $(X, M, *)$ is called an M -fuzzy metric space if X is an arbitrary (non empty) set, $*$ is a continuous t -norm and M is a fuzzy set on $X^3 \times (0, \infty)$ satisfying the following conditions for each $x, y, z, a \in X$ and $s, t > 0$:

- (i) $M(x, y, z, t) > 0$
- (ii) $M(x, y, z, t) = 1$ if and only if $x = y = z$
- (iii) $M(x, y, z, t) = M(P(x, y, z), t)$ where P is a permutation function.
- (iv) $M(x, y, a, t) * M(a, z, z, s) \leq M(x, y, z, t + s)$
- (v) $M(x, y, z, \cdot) : (0, \infty) \rightarrow (0, 1]$ is continuous.

For related definitions and examples of M fuzzy metric space the reader can refer [22] and [23].

Definition 7 ([29]). A 3-tuple $(X, Q, *)$ is called a Q -fuzzy metric space if X is an arbitrary (non empty) set, $*$ is a continuous t -norm and Q is a fuzzy set on $X^3 \times (0, \infty)$ satisfying the following conditions for each $x, y, z, a \in X$ and $s, t > 0$:

Q_1 : $Q(x, x, y, t) > 0$ and $Q(x, x, y, t) \leq Q(x, y, z, t)$ for all $x, y, z \in X$ with $z \neq y$

Q_2 : $Q(x, y, z, t) = 1$ if and only if $x = y = z$

Q_3 : $Q(x, y, z, t) = Q(P(x, y, z), t)$ (symmetry) where P is a permutation function.

Q_4 : $Q(x, a, a, t) * Q(a, y, z, s) \leq Q(x, y, z, t + s)$

Q_5 : $Q(x, y, z, \cdot) : (0, \infty) \rightarrow (0, 1]$ is continuous.

For details of Q -fuzzy metric space the reader can refer [29] and [20].

Definition 8 ([24]). Let X be a non empty set. An S - metric on X is a function $S : X^3 \rightarrow [0, \infty)$, that satisfies the following conditions for all $x, y, z, a \in X$.

- (i) $S(x, y, z) \geq 0$,
- (ii) $S(x, y, z) = 0$ if and only if $x = y = z$,
- (iii) $S(x, y, z) \leq S(x, x, a) + S(y, y, a) + S(z, z, a)$.

Then the function S is called an S metric and the pair (X, S) is called an S -metric space.

Remark 2. Every D^* -metric space and G -metric space is an S -metric space but in general, the converse is not true. The reader can refer to the examples and related definitions of S - metric space in [24].

Definition 9 ([13]). A t -norm T is said to be of H -type if the family of its iterates $\{T_n\}$ where n is natural number is given by $T^0(x) = 1$ and $T^n(x) = T(T^{n-1}(x), x)$ for all $n \geq 1$, is equicontinuous at $x = 1$. A trivial example of a t -norm of H -type is the minimum t -norm.

3. The N -fuzzy metric space

The concept of N -fuzzy metric space is defined as follows:

Definition 10. A triplet $(X, N, *)$ is an N -fuzzy metric space $(NFM S_s)$, if X is an arbitrary (non empty) set, $*$ is a continuous t -norm and N is a fuzzy set on $X^3 \times (0, \infty)$ satisfying the following conditions for all $x, y, z \in X$ and $r, s, t > 0$:

- (i) $N(x, y, z, t) > 0$
- (ii) $N(x, y, z, t) = 1$ if and only if $x = y = z$
- (iii) $N(x, y, z, r + s + t) \geq N(x, x, a, r) * N(y, y, a, s) * N(z, z, a, t)$
- (iv) $N(x, y, z, \cdot) : (0, \infty) \rightarrow (0, 1]$ is a continuous function.

Example 1. Let $X = R$ be a real line and S be an S -metric on X defined by

$$S(x, y, z) = |x - z| + |y - z|$$

or

$$S(x, y, z) = |y + z - 2x| + |y - z|.$$

Define $a * b * c = a b c$ for every $a, b, c \in [0, 1]$ and let N be the function on $X^3 \times (0, \infty)$ defined by $N(x, y, z, t) = \frac{t}{t + S(x, y, z)}$ for all $x, y, z \in X$ and $t > 0$.

Then $(R, N, *)$ is an N -fuzzy metric space, but it is not Q -fuzzy metric space and M -fuzzy metric space because N is not symmetric.

Example 2. Let $X = R$ be a real line and S be an S -metric as defined in above Example 1. Define $a * b * c = a b c$ for every $a, b, c \in [0, 1]$ and let N be the function on $X^3 \times (0, \infty)$ defined by $N(x, y, z, t) = [\exp[\frac{S(x, y, z)}{t}]]^{-1}$ for all $x, y, z \in X$ and $t > 0$.

Then $(R, N, *)$ is an N -fuzzy metric space, but it is not Q -fuzzy metric space and M -fuzzy metric space because N is not symmetric.

4. Topology of N -fuzzy metric space

Definition 11. Let $(X, N, *)$ be an N -fuzzy metric space. For $t > 0$, the open ball $B(x, r, t)$ with center $x \in X$ and radius $0 < r < 1$ is defined by $B(x, r, t) = \{y \in X : N(y, y, x, t) > 1 - r\}$.

The collection $\{B(x, r, t) : x \in X, 0 < r < 1, t > 0\}$ is a neighborhood system for topology τ on X induced by the N -fuzzy metric N .

Proposition 1. Let $(X, N, *)$ be an N -fuzzy metric space, then for all $x, y, z \in X$ and $t > 0$, we have $N(x, x, y, t) = N(y, y, x, t)$.

Proof. Since N -fuzzy metric is induced by S -metric and in S -metric space $S(x, x, y) = S(y, y, x)$ for all $x, y \in X$. Therefore in N -fuzzy metric space $N(x, x, y, t) = N(y, y, x, t)$ for all $x, y \in X$ and $t > 0$. ■

Lemma 1. $N(x, x, y, \cdot)$ is nondecreasing for all x, y in X .

Proof. Suppose that $N(x, x, y, t) > N(x, x, y, s)$ for some $0 < t < s$. Then

$$\begin{aligned} N(x, x, x, \frac{s}{2}) * N(x, x, x, \frac{s}{2} - t) * N(y, y, x, t) &\leq N(x, x, y, s) \\ &< N(x, x, y, t). \end{aligned}$$

By Definition 10, we have

$$N(x, x, x, \frac{s}{2}) = 1$$

and

$$N(x, x, x, \frac{s}{2} - t) = 1.$$

Thus

$$\begin{aligned} N(y, y, x, t) &\leq N(x, x, y, s) \\ N(x, x, y, t) &\leq N(x, x, y, s) \quad [\text{As } N(x, x, y, t) = N(y, y, x, t)] \end{aligned}$$

a contradiction. ■

Theorem 1. Every N -fuzzy metric space is Hausdorff.

Proof. Let $(X, N, *)$ be the N -fuzzy metric space. Let x, y be two distinct points of X . Then $0 < N(x, x, y, t) < 1$. Let $N(x, x, y, t) = r$ for some r , $0 < r < 1$. For each r_0 , $r < r_0 < 1$, we can find an r_1 such that $r_1 * r_1 * r_1 \geq r_0$. Now consider the open balls $B(x, 1 - r_1, \frac{t}{3})$ and $B(y, 1 - r_1, \frac{t}{3})$.

Clearly

$$B(x, 1 - r_1, \frac{t}{3}) \cap B(y, 1 - r_1, \frac{t}{3}) = \phi.$$

For if there exists

$$z \in B(x, 1 - r_1, \frac{t}{3}) \cap B(y, 1 - r_1, \frac{t}{3}).$$

Then

$$\begin{aligned} r = N(x, x, y, t) &\geq N(x, x, z, \frac{t}{3}) * N(x, x, z, \frac{t}{3}) * N(y, y, z, \frac{t}{3}) \\ &> r_1 * r_1 * r_1 \geq r_0 > r \end{aligned}$$

which is a contradiction. Therefore $(X, N, *)$ is Hausdorff. ■

Definition 12. A sequence $\{x_n\}$ in $(X, N, *)$ is converges to $x \in X$ if $N(x_n, x_n, x, t) \rightarrow 1$ or $N(x, x, x_n, t) \rightarrow 1$ as $n \rightarrow \infty$ for each $t > 0$. That is for each $\epsilon > 0$ and $t > 0$ there exists $n_0 \in \mathbf{N}$ such that for all $n \geq n_0$, $N(x_n, x_n, x, t) > 1 - \epsilon$ or $N(x, x, x_n, t) > 1 - \epsilon$.

Lemma 2. Let $(X, N, *)$ be an N -fuzzy metric space, where $*$ is minimum t -norm (H -type). Let $\{x_n\}$ be a sequence in X . If $\{x_n\}$ converges to x and $\{x_n\}$ also converges to y then $x = y$. That is the limit of $\{x_n\}$ if exists is unique.

Proof. Let $\{x_n\}$ converges to x and y . Then $N(x, x, x_n, r) \rightarrow 1$ as $n \rightarrow \infty$ for each $r > 0$ and $N(y, y, x_n, t - 2r) \rightarrow 1$ as $n \rightarrow \infty$ for each $t - 2r > 0$.

$$\begin{aligned} N(x, x, y, t) &\geq N(x, x, x_n, r) * N(x, x, x_n, r) * N(y, y, x_n, t - 2r) \\ &= 1 * 1 * 1 \quad \text{as } n \rightarrow \infty \\ &\rightarrow 1 \quad [\text{where } 1 * 1 * 1 = \min\{1, 1, 1\} \\ &\quad \text{by Definition 3, Example 2 (H-type)}]. \end{aligned}$$

■

Definition 13. Let $(X, N, *)$ be an N -fuzzy metric space and $\{x_n\}$ be a sequence in X is called Cauchy sequence. If for each $\epsilon > 0$ and $t > 0$ there exists $n_0 \in \mathbf{N}$ such that

$$N(x_n, x_n, x_m, t) > 1 - \epsilon$$

or

$$N(x_m, x_m, x_n, t) > 1 - \epsilon \quad \text{for all } n, m \geq n_0.$$

Definition 14. Let $(X, N, *)$ be an N -fuzzy metric space. If every Cauchy sequence in X is convergent in X , then X is called a complete N -fuzzy metric space.

Lemma 3. Let $(X, N, *)$ be an N -fuzzy metric space, where $*$ is minimum t -norm (H -type) and $\{x_n\}$ be a sequence in X . If $\{x_n\}$ converges to x , then $\{x_n\}$ is a Cauchy sequence.

Proof. For each $r, t > 0$ there is $\mathbf{N}$ such that $p \in \mathbf{N}$,

$$N(x_n, x_n, x, r) \rightarrow 1 \quad \text{as } n \rightarrow \infty$$

and

$$N(x_{n+p}, x_{n+p}, x, t - 2r) \rightarrow 1 \quad \text{as } n \rightarrow \infty \quad \text{for each } t - 2r > 0,$$

$$\begin{aligned}
N(x_n, x_n, x_{n+p}, t) &\geq N(x_n, x_n, x, r) * N(x_n, x_n, x, r) \\
&\quad * N(x_{n+p}, x_{n+p}, x, t - 2r) \\
&= 1 * 1 * 1 \quad \text{as } n \rightarrow \infty \\
&= 1 \quad [\text{where } 1 * 1 * 1 = \min\{1, 1, 1\} \\
&\quad \text{by Definition 3, Example 2 (H-type)}].
\end{aligned}$$

■

Therefore $\{x_n\}$ is a Cauchy sequence.

Remark 3. It is easy to prove that the induced N -fuzzy metric space $(X, N, *)$ is complete if and only if the S -metric space (X, S) is complete where $N(x, y, z, t) = \frac{t}{t+s(x, y, z)}$ for all $x, y, z \in X$ and $t \in (0, \infty)$.

Definition 15. Let $(X, N, *)$ and $(X', N', *)$ be N -fuzzy metric spaces. Then a function $f : X \rightarrow X'$ is said to be continuous at a point $x \in X$ if and only if it is sequentially continuous at x , that is whenever $\{x_n\}$ is convergent to x we have $\{fx_n\}$ is convergent to $f(x)$.

Lemma 4. Let $(X, N, *)$ be an N -fuzzy metric space. Let $\{x_n\}$ and $\{y_n\}$ be two sequences in X and suppose $x_n \rightarrow x, y_n \rightarrow y$ as $n \rightarrow \infty$ and $N(x, x, y, t_n) \rightarrow N(x, x, y, t)$ as $n \rightarrow \infty$. Then $N(x_n, x_n, y_n, t_n) \rightarrow N(x, x, y, t)$ as $n \rightarrow \infty$.

Proof. Since $\lim_{n \rightarrow \infty} x_n = x, \lim_{n \rightarrow \infty} y_n = y$ and $\lim_{n \rightarrow \infty} N(x, x, y, t_n) = N(x, x, y, t)$ there is $n_0 \in \mathbf{N}$ such that $|t - t_n| < \delta$ for $n \geq n_0$ and $\delta < \frac{t}{2}$. We know that $N(x, x, y, t)$ is nondecreasing with respect to t , so we have

$$\begin{aligned}
N(x_n, x_n, y_n, t_n) &\geq N(x_n, x_n, y_n, t - \delta) \\
&\geq N(x_n, x_n, x, \frac{\delta}{3}) * N(x_n, x_n, x, \frac{\delta}{3}) * N(y_n, y_n, x, t - \frac{5\delta}{3}) \\
&\geq N(x_n, x_n, x, \frac{\delta}{3}) * N(x_n, x_n, x, \frac{\delta}{3}) * N(y_n, y_n, y, \frac{\delta}{6}) \\
&\quad * N(y_n, y_n, y, \frac{\delta}{6}) * N(y, y, x, t - 2\delta)
\end{aligned}$$

and

$$\begin{aligned}
N(x, x, y, t + 2\delta) &\geq N(x, x, y, t_n + \delta) \\
&\geq N(x, x, x_n, \frac{\delta}{3}) * N(x, x, x_n, \frac{\delta}{3}) * N(y, y, x_n, t_n + \frac{\delta}{3}) \\
&\geq N(x, x, x_n, \frac{\delta}{3}) * N(x, x, x_n, \frac{\delta}{3}) * N(y, y, y_n, \frac{\delta}{6}) \\
&\quad * N(y, y, y_n, \frac{\delta}{6}) * N(x_n, x_n, y_n, t_n).
\end{aligned}$$

In view of Definition 12 and combining the arbitrariness of δ and the continuity of $N(x, x, y, \cdot)$ w.r. to t . For large enough n , we have

$$N(x, x, y, t) \geq N(x_n, x_n, y_n, t_n) \geq N(y, y, x, t),$$

$$N(x, x, y, t) \geq N(x_n, x_n, y_n, t_n) \geq N(x, x, y, t) \quad [\text{by Proposition 1}].$$

Consequently

$$\lim_{n \rightarrow \infty} N(x_n, x_n, y_n, t_n) \rightarrow N(x, x, y, t).$$

■

Lemma 5. *Let $(X, N, *)$ be an N -fuzzy metric space. If there exists $g \in (0, 1)$ such that $N(x, x, y, gt) \geq N(x, x, y, t)$ for all $x, y \in X$, $t > 0$ and*

$$\lim_{t \rightarrow \infty} N(x, y, z, t) = 1$$

then $x = y$.

Proof. Suppose that there exists $g \in (0, 1)$ such that $N(x, x, y, gt) \geq N(x, x, y, t)$ for all $x, y \in X$ and $t > 0$.

Then

$$N(x, x, y, t) \geq N(x, x, y, \frac{t}{g})$$

and so

$$N(x, x, y, t) \geq N(x, x, y, \frac{t}{g^n})$$

for positive integer n . Taking limit as $n \rightarrow \infty$, $N(x, x, y, t) \geq 1$ and hence $x = y$. ■

5. The various types of mapping in NFM spaces

Definition 16. *Let S and T maps from an NFM space $(X, N, *)$ into itself. The maps S and T are said to be compatible, if for all $t > 0$.*

$$\lim_{n \rightarrow \infty} N(STx_n, STx_n, TSx_n, t) = 1$$

whenever $\{x_n\}$ is a sequence in X such that $\lim_{n \rightarrow \infty} Sx_n = \lim_{n \rightarrow \infty} Tx_n = z$ for some $z \in X$.

Example 3. Let $X = [2, 20)$. For each $t \in (0, \infty)$ and for all $x, y, z \in X$, define

$$N(x, y, z, t) = \frac{t}{t + |x - z| + |y - z|}.$$

Clearly, $(X, N, *)$ is an N -fuzzy metric space, where $*$ is defined by $a * b * c = abc$. Let S and T be self- maps of X defined as

$$S(x) = \begin{cases} 2 & \text{if } x = 2 \text{ or } x > 5 \\ 6 & \text{if } 2 < x \leq 5 \end{cases}, \quad T(x) = \begin{cases} 2 & \text{if } x = 2 \text{ or } x > 5 \\ 12 & \text{if } 2 < x \leq 5 \\ \frac{x+1}{3} & \text{if } x > 5 \end{cases}.$$

Let sequence $\{x_n\}$ be defined as $x_n = 5 + \frac{1}{n}$, $n \geq 1$ then we have $\lim_{n \rightarrow \infty} Sx_n = \lim_{n \rightarrow \infty} Tx_n = 2$. Hence, S and T satisfy the property (E.A.). Also,

$$\lim_{n \rightarrow \infty} N(STx_n, STx_n, TSx_n, t) = \frac{t}{t + |2 - 2| + |2 - 2|} = \frac{t}{t + 0} = 1.$$

This shows that S and T are compatible.

Definition 17. Let S and T be maps from an NFM space $(X, N, *)$ into itself. The maps are said to be weakly compatible, if they commute at their coincidence points, that is, $Sz = Tz$ implies that $STz = T Sz$.

Definition 18. Let S and T maps from an NFM space $(X, N, *)$ into itself. The maps S and T are said to be semicompatible, if for all $t > 0$.

$$\lim_{n \rightarrow \infty} N(STx_n, STx_n, Tz, t) = 1$$

whenever $\{x_n\}$ is a sequence in X such that $\lim_{n \rightarrow \infty} Sx_n = \lim_{n \rightarrow \infty} Tx_n = z$ for some $z \in X$.

Note that the semicompatibility of the pair (S, T) , need not imply the semicompatibility of (T, S) .

The following is an example of a pair of self-maps (S, T) which is compatible but not semicompatible. Further, it is also seen that the semicompatibility of the pair (S, T) need not imply the semicompatibility of (T, S) .

Example 4. Let $X = [0, 1]$ and let $(X, N, *)$ be the N -fuzzy metric space with

$$N(x, y, z, t) = \left[\exp \frac{[|x - z| + |y - z|]}{t} \right]^{-1}$$

for all $x, y, z \in X$, $t > 0$.

Example 5. Define self- map S as follows:

$$S(x) = \begin{cases} x & \text{if } 0 \leq x < \frac{1}{2} \\ 1 & \text{if } x \geq \frac{1}{2} \end{cases}.$$

Let I be the identity map on X and $x_n = \frac{1}{2} - \frac{1}{n}$. Then, $\{Ix_n\} = \{x_n\} \rightarrow \frac{1}{2}$ and $\{Sx_n\} = \{x_n\} \rightarrow \frac{1}{2}$. Thus, $\{ISx_n\} = \{Sx_n\} = \frac{1}{2} \neq S\{\frac{1}{2}\}$. Hence (IS) is not semicompatible.

Again as (I, S) is commuting, it is compatible. Further, for any sequence $\{x_n\}$ in X such that $\{x_n\} \rightarrow x$ and $\{Sx_n\} \rightarrow x$, we have $\{SIx_n\} = \{Sx_n\} \rightarrow x = Ix$. Hence (SI) is always semicompatible.

Remark 4. The above example gives an important aspect of semicompatibility as the pair of self-maps (IS) is commuting, hence it is weakly commuting, compatible and weak compatible yet it is not semicompatible. Further, it is to be noted that the pair (S, I) is semicompatible but (I, S) is not semicompatible here.

The following is an example of a pair of self maps (A, S) which is semicompatible but not compatible.

Example 6. Let $X = [0, 2]$ and $(X, N, *)$ be an N -fuzzy metric space, where the definition of $*$ and N are same as defined in Example 3. Define self-maps A and S on X as follows

$$A(x) = \begin{cases} 2 & \text{if } 0 \leq x \leq 1 \\ \frac{x}{2} & \text{if } 1 < x \leq 2 \end{cases}, \quad S(x) = \begin{cases} 2 & \text{if } x = 1 \\ \frac{x+3}{5} & \text{otherwise} \end{cases}$$

and $x_n = 2 - \frac{1}{2n}$, $n \geq 1$. Then we have $S(1) = A(1) = 2$ and $S(2) = A(2) = 1$. Also $SA(1) = AS(1) = 1$ and $SA(2) = AS(2) = 2$. Thus (A, S) is weak compatible. Again, $Ax_n = 1 - \frac{1}{4n}$, $Sx_n = 1 - \frac{1}{10n}$. Thus, $Ax_n \rightarrow 1$, $Sx_n \rightarrow 1$. Hence $u = 1$

Further,

$$SAx_n = \frac{4}{5} - \frac{1}{20n}, \quad ASx_n = 2.$$

Now,

$$\lim_{n \rightarrow \infty} N(ASx_n, ASx_n, Su, t) = \lim_{n \rightarrow \infty} N(2, 2, 2, t) = 1,$$

$$\begin{aligned} \lim_{n \rightarrow \infty} N(ASx_n, ASx_n, SAx_n, t) &= \lim_{n \rightarrow \infty} N\left(2, 2, \frac{4}{5} - \frac{1}{20n}, t\right) \\ &= \frac{t}{t + \left|2 - \frac{4}{5}\right| + \left|2 - \frac{4}{5}\right|} \\ &= \frac{t}{t + \frac{12}{5}} < 1 \quad \forall t > 0. \end{aligned}$$

Hence (A, S) is semicompatible but it is not compatible.

Definition 19. Let S and T be two self-maps of an NFM space $(X, N, *)$. We say that S and T satisfy the property (E.A.) if there exists a sequence $\{x_n\}$ such that $\lim_{n \rightarrow \infty} Sx_n = \lim_{n \rightarrow \infty} Tx_n = z$ for some $z \in X$.

Remark 5. Note that the weakly compatible and property (E.A.) are independent to each other. See the following example.

Example 7. Let $(X, N, *)$ be an N -Fuzzy metric space where $X = [0, 1]$ and N is defined as in Example 3. Define $S, T : X \rightarrow X$ by

$$S(x) = 1 - x, \text{ if } x \in [0, \frac{1}{2}] \text{ and } S(x) = 0, \text{ if } x \in (\frac{1}{2}, 1]$$

$$T(x) = \frac{1}{2}, \text{ if } x \in [0, \frac{1}{2}] \text{ and } T(x) = \frac{3}{4}, \text{ if } x \in (\frac{1}{2}, 1].$$

Then, for the sequence $\{x_n\} = \{\frac{1}{2} - \frac{1}{n}\}$, $n \geq 2$, we have

$$\lim_{n \rightarrow \infty} S(\frac{1}{2} - \frac{1}{n}) = \lim_{n \rightarrow \infty} \frac{1}{2} + \frac{1}{n} = \frac{1}{2} = \lim_{n \rightarrow \infty} T(\frac{1}{2} - \frac{1}{n}).$$

Thus the pair (S, T) satisfies property (E.A.). Further, S and T are weakly compatible since $x = \frac{1}{2}$ is their unique coincidence point and $ST(\frac{1}{2}) = S(\frac{1}{2}) = T(\frac{1}{2}) = TS(\frac{1}{2})$. We further observe that

$$\begin{aligned} \lim_{n \rightarrow \infty} N\left(ST(\frac{1}{2} - \frac{1}{n}), ST(\frac{1}{2} - \frac{1}{n}), TS(\frac{1}{2} - \frac{1}{n})\right) \\ = \lim_{n \rightarrow \infty} \frac{t}{t + 2 \mid ST(\frac{1}{2} - \frac{1}{n}) - TS(\frac{1}{2} - \frac{1}{n}) \mid} \\ = \frac{t}{t + 2 \mid \frac{1}{2} - \frac{3}{4} \mid} = \frac{t}{t + \frac{1}{2}} \neq 1, \end{aligned}$$

showing that the pair (S, T) is noncompatible.

Example 8. Let $(X, N, *)$ be an N -fuzzy metric space where $X = R^+$ with t -norm defined by $a * b * c = a.b.c$ for all $a, b, c \in [0, 1]$ and $N(x, y, z, t) = \frac{t}{t + |x-y| + |y-z| + |z-x|}$ for all $t > 0$ and $x, y, z \in X$. Define $S, T : X \rightarrow X$ by $S(x) = 0$, if $0 < x \leq 1$ and $S(x) = 1$, if $x > 1$ or $x = 0$; and $T(x) = [x]$, the greatest integer that is less than or equal to $x \forall x \in X$.

Consider a sequence $\{x_n\} = \{1 + \frac{1}{n}\}_{n \geq 2}$ in $(1, 2)$, then we have

$$\lim_{n \rightarrow \infty} Sx_n = 1 = \lim_{n \rightarrow \infty} Tx_n.$$

Similarly, for the sequence $\{y_n\} = \{1 - \frac{1}{n}\}_{n \geq 2}$ in $(0, 1)$, we have

$$\lim_{n \rightarrow \infty} Sy_n = 0 = \lim_{n \rightarrow \infty} Ty_n.$$

Thus the pair (S, T) satisfies the property (E.A.). However S and T are not weakly compatible; as each $u_1 \in (0, 1)$ and $u_2 \in (1, 2)$ are coincidence points of S and T , where they do not commute. Moreover, they commute at $x = 0, 1, 2, \dots$, but none of these points are coincidence points of S and T . Further, (S, T) is non compatible. Hence property (E.A.) does not imply weak compatibility.

Remark 6. From the Definition 16, it is inferred that two self maps S and T on NFM space $(X, N, *)$ are noncompatible iff there exists at least one sequence $\{x_n\}$ in X such that $\lim_{n \rightarrow \infty} Sx_n = \lim_{n \rightarrow \infty} Tx_n = z$ for some $z \in X$, but for some $t > 0$ either $\lim_{n \rightarrow \infty} N(STx_n, STx_n, TSx_n, t) \neq 1$ or the limit does not exist. Therefore, any two noncompatible self- maps of $(X, N, *)$ satisfy the property $(E.A)$ from the Definition 19. But the Example 3 shows that two maps satisfy the property $(E.A)$ need not be noncompatible.

Note. For detail study of compatible maps, noncompatible maps, weak compatible, semicompatible maps and property $(E.A.)$ in metric space, the reader can refer [28], [18] and [2].

5. Pseudo N -fuzzy metric spaces

Definition 20. A 3-tuple $(X, N, *)$ is said to be Pseudo N -fuzzy metric space if X is an arbitrary (non empty) set, $*$ is a continuous t -norm and N is a fuzzy set on $X^3 \times (0, \infty)$ satisfying the following conditions.

- $P_1 : \forall x, y, z \in X \text{ and } \forall t > 0 \ N(x, y, z, t) > 0$
- $P_2 : \forall x, y, z \in X \text{ and } \forall t > 0 \ N(x, y, z, t) = 1 \text{ if } x = y = z$
- $P_3 : \forall x, y, z, a \in X \text{ and } \forall r, s, t > 0 \ N(x, y, z, r + s + t) \geq N(x, x, a, r) * N(y, y, a, s) * N(z, z, a, t)$
- $P_4 : \forall x, y, z \in X, \ N(x, y, z, .) : (0, \infty) \rightarrow (0, 1] \text{ is continuous.}$

Remark 7. Clearly every N -fuzzy metric space is a Pseudo N -fuzzy metric space but converse is not true [see the following Example 9].

Example 9. Consider R with the usual metric. Let $X = \{\{x_n\} : \{x_n\} \text{ is convergent in } R\}$. Define $a * b * c = a.b.c$ for all $a, b, c \in [0, 1]$ and

$$N(x_n, y_n, z_n, t) = \left[\exp \frac{\lim [|x_n - z_n| + |y_n - z_n|]}{t} \right]^{-1}.$$

Clearly $(X, N, *)$ is a Pseudo N -fuzzy metric space but not N -fuzzy metric space. To see this:

Let $\{x_n\} = \frac{1}{n}$, $\{y_n\} = \frac{2}{n}$ and $\{z_n\} = \frac{3}{n}$. Then $x_n \neq y_n \neq z_n$ for all $x_n, y_n, z_n \in X$ but $N(x_n, y_n, z_n, t) = 1$.

A class of implicit relation [1]. Let Φ be the set of all real continuous functions $\Phi : [R^+]^4 \rightarrow R$, nondecreasing in first argument and satisfying the following conditions.

(1) (i) For $u, v \geq 0$, $\Phi(u, v, v, u) \geq 0$ or $\Phi(u, v, u, v) \geq 0$ implies $u \geq v$

(2) (ii) $\Phi(u, v, 1, 1) \geq 0$ implies that $u \geq 1$.

Example 10. 1. Define $\phi(t_1, t_2, t_3, t_4) = 15t_1 - 13t_2 + 5t_3 - 7t_4$. Then $\phi \in \Phi$

2. Define $\phi(t_1, t_2, t_3, t_4) = 14t_1 - 12t_2 + 6t_3 - 8t_4$. Then $\phi \in \Phi$

6. Application in fixed point theory

As an application of weak compatible maps and the property(EA), we prove the fixed point theorem of Irshad Aalam et.al [1] in NFM space.

Theorem 2. *Let E, F, S and T be self maps of an NFM space $(X, N, *)$ satisfying the following conditions:*

$$(3) \quad E(X) \subseteq T(X), \quad F(X) \subseteq S(X);$$

$$(4) \quad (E, S) \text{ and } (F, T) \text{ are weakly compatible pairs};$$

$$(5) \quad (E, S) \text{ or } (F, T) \text{ satisfies the property } (E.A.);$$

For some $\phi \in \Phi$, there exist $k \in (0, 1)$ such that for all $x, y, \in X, t > 0$

$$(6) \quad \phi \left(N(Ex, Ex, Fy, kt), N(Sx, Sx, Ty, t), \right. \\ \left. N(Ex, Ex, Sx, t), N(Fy, Fy, Ty, t) \right) \geq 0.$$

If the range of one of the maps E, F, S or T is a complete subspace of X , then E, F, S and T have a unique common fixed point in X .

Proof. If the pair (F, T) satisfies the property $(E.A.)$ then there exists a sequence $\{x_n\}$ such that $Fx_n \rightarrow z$ and $Tx_n \rightarrow z$ for some $z \in X$ as $n \rightarrow \infty$.

Since $F(X) \subseteq S(X)$, there exists in X a sequence $\{y_n\}$ such that $Fx_n = Sy_n$. Hence $Sy_n \rightarrow z$ as $n \rightarrow \infty$.

Now we claim that $Ey_n \rightarrow z$ as $n \rightarrow \infty$. Suppose $Ey_n \rightarrow w$ ($\neq z$) $\in X$, then by (6), we have

$$\phi \left(N(Ey_n, Ey_n, Fx_n, kt), N(Sy_n, Sy_n, Tx_n, t), \right. \\ \left. N(Ey_n, Ey_n, Sy_n, t), N(Fx_n, Fx_n, Tx_n, t) \right) \geq 0$$

that is,

$$\phi \left(N(Ey_n, Ey_n, Fx_n, kt), N(Fx_n, Fx_n, Tx_n, t), \right. \\ \left. N(Ey_n, Ey_n, Fx_n, t), N(Fx_n, Fx_n, Tx_n, t) \right) \geq 0.$$

As ϕ is nondecreasing in the first argument, we have

$$\phi\left(N(Ey_n, Ey_n, Fx_n, t), N(Fx_n, Fx_n, Tx_n, t),\right. \\ \left. N(Ey_n, Ey_n, Fx_n, t), N(Fx_n, Fx_n, Tx_n, t)\right) \geq 0.$$

Using (1), we get $N(Ey_n, Ey_n, Fx_n, t) \geq N(Fx_n, Fx_n, Tx_n, t)$. Letting $n \rightarrow \infty$, $N(w, w, z, t) \geq 1$ for all $t > 0$. Hence, $N(w, w, z, t) = 1$ Thus $w = z$. This shows that $Ey_n \rightarrow z$ as $n \rightarrow \infty$.

Suppose that $S(X)$ is a complete subspace of X . Then $z = Su$ for some $u \in X$. Subsequently, we have $Ey_n \rightarrow Su, Fx_n \rightarrow Su, Tx_n \rightarrow Su$ and $Sy_n \rightarrow Su$ as $n \rightarrow \infty$.

By (6), we have

$$\phi\left(N(Eu, Eu, Fx_n, kt), N(Su, Su, Tx_n, t),\right. \\ \left. N(Eu, Eu, Su, t), N(Fx_n, Fx_n, Tx_n, t)\right) \geq 0.$$

Letting $n \rightarrow \infty$

$$\phi\left(N(Eu, Eu, Su, kt), 1, N(Eu, Eu, Su, t), 1\right) \geq 0.$$

As ϕ is nondecreasing in the first argument, we have

$$\phi\left(N(Eu, Eu, Su, t), 1, N(Eu, Eu, Su, t), 1\right) \geq 0.$$

Using (1), we get $N(Eu, Eu, Su, t) \geq 1$ for all $t > 0$. Hence, $N(Eu, Eu, Su, t) = 1$. Thus, $Eu = Su$.

The weak compatibility of E and S implies that $ESu = SEu$ and then $EEu = ESu = SEu = SSu$.

On the other hand, since $E(X) \subseteq T(X)$, there exists a $v \in X$ such that $Eu = Tv$. We show that $Tv = Fv$. By (6), we have

$$\phi\left(N(Eu, Eu, Fv, kt), N(Su, Su, Tv, t),\right. \\ \left. N(Eu, Eu, Su, t), N(Fv, Fv, Tv, t)\right) \geq 0$$

that is,

$$\phi\left(N(Tv, Tv, Fv, kt), 1, 1, N(Fv, Fv, Tv, t)\right) \geq 0.$$

As ϕ is nondecreasing in the first argument, we have

$$\phi\left(N(Tv, Tv, Fv, t), 1, 1, N(Fv, Fv, Tv, t)\right) \geq 0 \\ [As\ N(x, x, y, t) = N(y, y, x, t)].$$

Using (1), we get $N(Tv, Tv, Fv, t) \geq 1$ for all $t > 0$. Hence, $N(Tv, Tv, Fv, t) = 1$. Thus, $Fv = Tv$.

This implies $Eu = Su = Tv = Fv$. The weak compatibility of F and T implies that $FTv = TFv$ and then $TTv = TFv = FTv = FFv$.

Now, we will show that Eu is a common fixed point of E, F, S and T . In view of (6) it follows

$$\phi\left(N(EEu, EEu, Fv, kt), N(SEu, SEu, Tv, t),\right. \\ \left.N(EEu, EEu, SEu, t), N(Fv, Fv, Tv, t)\right) \geq 0$$

that is,

$$\phi\left(N(EEu, EEu, Eu, kt), N(EEu, EEu, Eu, t), 1, 1\right) \geq 0.$$

As ϕ is nondecreasing in the first argument, we have

$$\phi\left(N(EEu, EEu, Eu, t), N(EEu, EEu, Eu, t), 1, 1\right) \geq 0$$

Using (2), we get $N(EEu, EEu, Eu, t) \geq 1$ for all $t > 0$. Hence, $N(EEu, EEu, Eu, t) = 1$. Thus, $EEu = Eu$.

Therefore, $Eu = EEu = SEu$ and Eu is a common fixed point of E and S . Similarly, we prove that Fv is a common fixed point of F and T . Since $Eu = Fv$, we conclude that Eu is a common fixed point of E, F, S and T . The proof is similar when $T(X)$ is assumed to be a complete subspace of X . The cases in which $E(X)$ or $F(X)$ is a complete subspace of X are similar to the cases in which $T(X)$ or $S(X)$ respectively, is complete since $E(X) \subseteq T(X), F(X) \subseteq S(X)$. If $Eu = Fu = Tu = Su = u$ and $Ev = Fv = Sv = Tv = v$ then (6) gives

$$\phi\left(N(Eu, Eu, Fv, kt), N(Su, Su, Tv, t), N(Eu, Eu, Su, t),\right. \\ \left.N(Fv, Fv, Tv, t)\right) \geq 0$$

that is,

$$\phi\left(N(u, u, v, kt), N(u, u, v, t), 1, 1\right) \geq 0.$$

Using (2), we get $N(u, u, v, t) \geq 1$ for all $t > 0$. Hence, $N(u, u, v, t) = 1$. Thus, $u = v$. Therefore, the common fixed point is unique. ■

The following example illustrates our result.

Example 11. Let $(X, N, *)$ be an N -fuzzy metric space as defined in Example 3. Define $E, F, S, T : X \rightarrow X$ by

$$E(x) = \begin{cases} 2 & \text{if } x = 2 \\ 3 & \text{if } 2 < x < 20 \end{cases}, \quad F(x) = \begin{cases} 2 & \text{if } x = 2 \\ 7 & \text{if } 2 < x < 20 \end{cases},$$

$$S(x) = \begin{cases} 2 & \text{if } x = 2 \\ 6 & \text{if } 2 < x \leq 10 \\ (x-7) & \text{if } 10 < x < 20 \end{cases}, \quad T(x) = \begin{cases} 2 & \text{if } x = 2 \\ 3 & \text{if } 2 < x \leq 10 \\ (x-3) & \text{if } 10 < x < 20 \end{cases}.$$

Then E , F , S and T satisfy all the conditions of Theorem (2) with $k \in (0, 1)$ and have a unique common fixed point $x = 2$. Clearly (E, S) and (F, T) are weakly compatible since they commute at their coincidence points. Let sequence $\{x_n\}$ be defined as $x_n = 10 + \frac{1}{n}$, $n \geq 1$, then we have $\lim_{n \rightarrow \infty} Fx_n = \lim_{n \rightarrow \infty} Tx_n = 7$. Hence, F and T satisfy the property (E.A.).

Conclusion. In the present study, we introduced the notion of N -fuzzy metric space, which generalized various fuzzy metric spaces like S -fuzzy metric space, M -fuzzy metric space, Q -fuzzy metric space, fuzzy 2-metric space and fuzzy metric space. We also prove a fixed point theorem using implicit relation, weak compatibility and the property (E.A.). This theorem extend the well known result of I. Aalam [1] et. al in new structure. Also, our result does not require either the completeness of the whole space or continuity of the maps.

We have also defined few more definitions like Pseudo- NFM space and Compatible maps, Weak compatible maps, Semi compatible maps and the property (E.A.) in NFM space.

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