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CONGESTION PROBABILITIES IN ERLANG-ENGSET MULTIRATE LOSS MODELS UNDER THE MULTIPLE FRACTIONAL CHANNEL RESERVATION POLICY

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Abstract. A communication link that accommodates different service-classes whose calls have different bandwidth requirements and compete for the available bandwidth under the Multiple Fractional Channel Reservation (MFCR) policy is considered. The MFCR policy allows the reservation of real number of channels in order to favor high speed calls. Two call arrival processes are studied: i) the Poisson (random) process and ii) the quasi-random process. In the first case, calls come from an infinite number of sources while in the second case calls are generated by a finite number of sources. To determine call blocking probabilities for Poisson arriving calls, recursive formulas are proposed based on reverse transition rates. To determine time and call congestion probabilities for quasi-random arriving calls, recursive formulas are proven based on the fact that the steady state probabilities cannot be described by a product form solution. The accuracy of the new formulas is verified through simulation.

1 Introduction

Modern communication networks require Quality of Service (QoS) mechanisms in order to provide the necessary bandwidth needed by multirate service-classes. Considering multirate traffic in a link, modeled as a loss system (i.e., queueing of calls is not allowed), which accommodates service-classes with different QoS requirements, such a QoS mechanism is a bandwidth sharing policy. The QoS assessment of service systems under a bandwidth sharing policy is accomplished through multirate teletraffic loss/queueing models [32].

The simplest bandwidth sharing policy is the Complete Sharing (CS) policy, where a new call is accepted in the link if there exists available bandwidth. Otherwise, call blocking occurs. The main call-level multirate loss model that adopts the CS policy is the Erlang Multirate Loss Model (EMLM) [18, 28]. In the EMLM, calls arrive in the link according to a Poisson process, require a certain amount of bandwidth and have generally distributed service times. The fact that the link occupancy distribution and Call Blocking Probabilities (CBP) are calculated via the accurate and recursive

Kaufman-Roberts formula ([18, 28]) has led to various extensions of the EMLM either in wired (e.g., [10, 12, 13, 15, 16, 19, 21, 26, 31, 33, 34, 36, 39]) or wireless (e.g., [5, 7, 17, 22, 27, 35, 37, 38, 44, 47]), satellite (e.g., [45, 46]) and optical networks (e.g., [3, 4, 6, 41–43]).

The dominant disadvantage of the CS policy is that it does not provide a specific QoS to service-classes. In addition, it is unfair to service-classes of high bandwidth requirements since it results in higher CBP compared to CBP of service-classes with low bandwidth requirements. A policy that guarantees QoS to new calls is the Bandwidth Reservation (BR) policy [32]. In the BR policy, an integer number of channels is reserved in favour of high speed calls. The BR policy achieves equalization of CBP among different service-classes by increasing the CBP of low speed calls (e.g., [9, 12, 21, 25, 29, 31, 34, 40]).

Another bandwidth sharing policy that deserves attention is the threshold policy. In the threshold policy, the number of in-service calls of certain service-classes cannot exceed certain thresholds. This means that a new call will be blocked, even if available bandwidth exists, when the threshold (different for each service-class) is exceeded after the acceptance of the call. For applications of the threshold policy or a combination of the threshold policy with the BR policy the interested reader may resort to [45, 46] and [25], respectively.

In this paper, we study the Multiple Fractional Channel Reservation (MFCR) policy which extends the BR policy [5]. The MFCR policy allows the reservation of a real number of channels. More specifically, $R_{r,k}$ channels are reserved in favour of all service-classes apart from *k*. Such a reservation is achieved by reserving $\lfloor R_{r,k} \rfloor + 1$ channels with probability $z_k = R_{r,k} - \lfloor R_{r,k} \rfloor$or $\lfloor R_{r,k} \rfloor$ channels with probability $1 - z_k$, where $\lfloor R_{r,k} \rfloor$ refers to the largest integer not exceeding $R_{r,k}$. In [5], the case of Poisson (random) traffic is considered. The model of [5] is named herein Random MFCR (R-MFCR). In the R-MFCR, the steady state distribution has no Product Form Solution (PFS) since local balance is destroyed between states that are adjacent. This means that the link occupancy distribution and other performance measures including CBP can be determined by approximate formulas which are recursive. Depending on the selection of the R-MFCR parameters, the model of [5] can provide the same CBP results with the model of [29], whereby a recursive formula is proposed for the CBP calculation in a multirate loss system of Poisson arrivals assuming the existence of the BR policy.

The accuracy of the analytical results provided by the model of [5] can be improved (compared to simulation) by introducing Reverse Transition Rates (RTR) that appear in the reservation space of calls of certain service-classes. The introduction of RTR has been initially studied in [31], in the case of Roberts' model [29]. Various extensions of [31], (see e.g., [11, 12, 20, 23]) show that such transition rates improve the analytical CBP results especially when the reservation space becomes large, a situation that appears when equalization of CBP is required.

In addition, the R-MFCR model is extended by assuming that calls are generated by finite sources. This is the well-known quasi-random process (e.g., [1, 2, 6, 8, 14, 24, 33, 48]). The springboard for the analysis of quasi-random traffic in a multirate loss system can be considered the Engset Multirate Loss Model (EnMLM) proposed in [33]. The model of [33] is named herein EnMLM since it results, for one service-class, in the same congestion probabilities with the Engset formula. To this end, the works of [33] and [5] are extended to the Quasi-Random MFCR model (QR-MFCR), which is new and describes a multirate loss system of *K* service-classes whose calls have fixed bandwidth requirements and a service time which is exponentially distributed. The new model does not have a PFS due to the MFCR policy. However, recursive formulas are proposed for the calculation of Time Congestion (TC) probabilities, Call Congestion

(CC) probabilities and link utilization. The accuracy of these formulas is highly satisfactory compared to simulation.

This paper is organized as follows: In Section 2, the R-MFCR model of [5] is reviewed. In Section 3, the method of RTR under the MFCR policy is presented, and formulas are provided for the link occupancy and CBP determination. In Section 4, the QR-MFCR model is proposed and recursive formulas are proven for the calculation of the link occupancy distribution and consequently TC and CC probabilities as well as link utilization. In Section 5, analytical CBP results are presented for the MFCR/RTR and the R-MFCR models. Based on the analytical and simulation results it can be shown that the MFCR/RTR model is more accurate compared to the R-MFCR. Furthermore, analytical and simulation TC probabilities results for the QR-MFCR model and analytical TC probabilities for the models of [18, 28] and [5] are presented. Section 6 presents the conclusion and possible future directions.

2 The R-MFCR Model

Consider a single link of capacity C channels. Calls of K service-classes compete for the available channels via the MFCR policy. A call of service class k ($k = 1, \ldots, K$) arrives in the link according to a Poisson process with parameter λ_k, requests c_k channels and has an MFCR parameter $R_{r,k}$. The latter refers to the real number of channels reserved in favour of all service-classes except from k. To reserve $R_{r,k}$ channels, $\lfloor R_{r,k} \rfloor + 1$ channels are reserved with probability z_k and $\lfloor R_{r,k} \rfloor$ channels with probability $1-z_k$.

Consider now an arriving call of service-class k that finds j occupied channels ($j = 0, 1, \ldots, C$) upon its arrival in the system. Let $x = C - j$ be the number of free channels. Then, to accept or reject this new call the following admission control cases are examined: a) if $x - \lfloor R_{r,k} \rfloor > c_k$, then the call is allowed to enter the system, b) if $x - \lfloor R_{r,k} \rfloor = c_k$, then the system accepts the new call with probability $1-z_k$ and c) if $x - \lfloor R_{r,k} \rfloor < c_k$, then the call is blocked and lost. Accepted service-class k calls receive an exponentially distributed service time of mean μ_k^{-1}.

Let $G(j)$ be the link occupancy distribution. Then, an approximate but recursive formula is proposed in [5] for the determination of the unnormalized values of $G(j)$'s:

$$G(j) = \begin{cases} 1 & \text{for } j = 0 \\ \frac{1}{j} \sum_{k=1}^{K} l_k(j-c_k) c_k G(j-c_k) & \text{for } j = 1, \ldots, C \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

where:

$$l_k(j-c_k) = \begin{cases} l_k & \text{for } j < C - \lfloor R_{r,k} \rfloor \\ (1-z_k) l_k & \text{for } j = C - \lfloor R_{r,k} \rfloor \\ 0 & \text{for } j > C - \lfloor R_{r,k} \rfloor \end{cases} \quad (2)$$

and $l_k = \lambda_k \mu_k^{-1}$ is the total offered traffic-load of service-class k calls (in erl).

Having determined $G(j)$'s the CBP of service-class k calls, B_k, is calculated via [5]:

$$B_k = \sum_{j=C-c_k-\lfloor R_{r,k} \rfloor+1}^{C} \frac{G(j)}{G} + z_k \frac{G(C-c_k-\lfloor R_{r,k} \rfloor)}{G} \quad (3)$$

where $G = \sum_{j=0}^{C} G(j)$ is the normalization constant.

In addition, the link utilization, U and the average number of service-class k calls in state j, $y_k(j)$, can be calculated via (4), (5), respectively:

$$U = \sum_{j=1}^{C} j G^{-1} G(j) \quad (4)$$

$$y_k(j) = \begin{cases} \frac{l_k G(j-c_k)}{G(j)} & \text{for } j < C - \lfloor R_{r,k} \rfloor \\ \frac{(1-z_k) l_k G(j-c_k)}{G(j)} & \text{for } j = C - \lfloor R_{r,k} \rfloor \\ 0 & \text{for } j > C - \lfloor R_{r,k} \rfloor \end{cases} \quad (5)$$

According to (5), $y_k(j)$=0 in the reservation space of service-class k given by the states: $j = C - \lfloor R_{r,k} \rfloor + 1, \ldots, C$. Clearly, the larger the value of $\lfloor R_{r,k} \rfloor$, the larger the reservation space becomes. This means, that in many states j, the values of $y_k(j)$'s will be zero (based on (5)). According to Section 3, the introduction of RTR in the reservation space of a service-class k results in positive values for $y_k(j)$'s, a fact that improves the CBP accuracy, compared to simulation.

Assuming a link under the BR policy, where an integer number of channels, R_k, is reserved, the link occupancy distribution is determined by (1), the link utilization by (4) while (2), (3) and (5) take the form of (6), (7) and (8), respectively [29]:

$$l_k(j-c_k) = \begin{cases} l_k & \text{for } j \leq C-R_k \\ 0 & \text{for } j > C-R_k \end{cases} \tag{6}$$

$$B_k = \sum_{j=C-c_k-R_k+1}^{C} G^{-1}G(j) \tag{7}$$

$$y_k(j) = \begin{cases} \frac{l_k G(j-c_k)}{G(j)} & \text{for } j \leq C-R_k \\ 0 & \text{for } j > C-R_k \end{cases} \tag{8}$$

In the case of the CS policy, $G(j)$'s can be determined by the classical Kaufman-Roberts recursion (9), CBP by (10) and the values of $y_k(j)$ by (11) ([18, 28]):

$$G(j) = \begin{cases} 1 & \text{for } j = 0 \\ \frac{1}{j}\sum_{k=1}^{K} l_k c_k G(j-c_k) & \text{for } j = 1,\ldots,C \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

$$B_k = \sum_{j=C-c_k+1}^{C} G^{-1}G(j) \tag{10}$$

$$y_k(j) = \begin{cases} \frac{l_k G(j-c_k)}{G(j)} & \text{for } j \leq C \\ 0 & \text{for } j > C \end{cases} \tag{11}$$

3 The RTR Method in the MFCR Policy

To apply the method of RTR, consider a link of C=3 channels and K = 2 service-classes accommodated in the link. Let the values of offered traffic-loads be l_1, l_2, while c_1=1 channel and c_2=2 channels. Calls of the 1st service-class have an MFCR parameter of $R_{r,1}$ = 1.2.

Fig. 1 shows the corresponding Markov chain of this system where each of the four states expresses the occupied link channels and $j = 0, 1, 2, 3$.

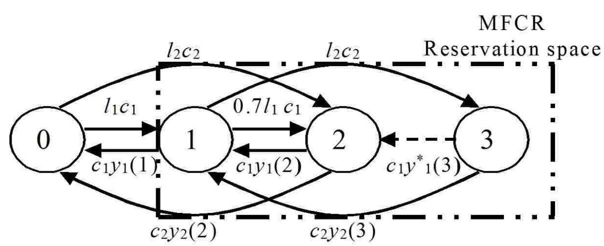

Fig. 1: The Markov chain of the example

According to (5), $y_1(3)$ = 0. Assume now that $j = 1$ when a call of the 2nd service-class arrives in the system. This means that a 1st service-class call is already in-service. The new call is accepted in the system and the system's state becomes $j = 3$. This means that from state $j = 3$ to $j = 2$ there should be a transition rate $c_1 y_1^*(3)$, where $y_1^*(3)$ denotes the (unknown) average number of 1st service-class calls in state $j = 3$. This transition rate is illustrated in Fig. 1 with a dashed arrow.

To determine the values of $y_k^*(j)$, an approximate formula is proposed ($y_k(j) \equiv y_k^*(j)$):

$$y_k(j) = \begin{cases} \frac{l_k G(j-c_k)}{G(j)} & \text{for } j < C-\lfloor R_{r,k} \rfloor \\ \sum_{i=1, i\neq k}^{K} y_k(j-c_i) w_{k,i}(j) & \text{for } j \geq C-\lfloor R_{r,k} \rfloor \end{cases} \tag{12}$$

where: $G(j)'s$ will be determined by (9) when $j < C -$

$\lfloor R_{r,k} \rfloor$, while $w_{k,i}(j)$ is a weight calculated by:

$$w_{k,i}(j) = \begin{cases} \frac{l_i c_i}{\sum\limits_{x=1, x \neq k}^{K} l_x c_x} & \text{for } j < C - \lfloor R_{r,i} \rfloor \\ \frac{(1-z_i) l_i c_i}{\sum\limits_{x=1, x \neq k}^{K} l_x c_x} & \text{for } j = C - \lfloor R_{r,i} \rfloor \end{cases} \quad (13)$$

where:

$$l_x c_x = \begin{cases} l_x c_x & \text{for } j < C - \lfloor R_{r,x} \rfloor \\ (1-z_x) l_x c_x & \text{for } j = C - \lfloor R_{r,x} \rfloor \end{cases} \quad (14)$$

The weight $w_{k,i}(j)$ refers to the proportion of the RTR $y_k^*(j)$ transferred in state j by a service-class $i \neq k$ call. Various weights have been tried instead of (13) but in most cases the weights determined via (13) provide highly satisfactory CBP results compared to the corresponding simulation CBP results.

Having calculated $y_k^*(j)$, the modified $G(j)$'s are given by:

$$G(j) = \begin{cases} 1 & \text{for } j = 0 \\ \frac{1}{j^*} \sum\limits_{k=1}^{K} l_k(j-c_k) c_k G(j-c_k) & \text{for } j = 1, \dots, C \\ 0 & \text{otherwise} \end{cases} \quad (15)$$

where: $j^* = \sum_{k=1}^{K} c_k y_k^*(j)$ and $l_k(j-c_k)$ is given by (2).

The CBPs can now be obtained via (3). Since the method of RTR increases the complexity of the calculations but does not necessarily improve the CBP accuracy (based on the corresponding CBP simulation results) when quasi-random traffic is considered, it is not adopted in the QR-MFCR model, presented in the following section.

4 The Proposed QR-MFCR Model

Consider a link of C channels that accommodates K service-classes. Calls of service class k ($k = 1, \dots, K$) come from a finite source population N_k, request c_k channels and compete for the available channels under the MFCR policy. Let $\lambda_{k,\mathrm{f}} = (N_k - n_k) v_k$ be the arrival rate of the idle sources of service-class k where n_k, v_k refer to the in-service calls and the arrival rate of idle sources, respectively. The offered traffic-load per service-class k idle source is $l_{k,\mathrm{f}} = v_k / \mu_k$ (in erl).

Fig. 2 presents the transition rates (in steady state) of the proposed model. According to Fig.2, the global balance equation for state $\mathbf{n} = (n_1, n_2, \dots, n_k, \dots, n_K)$, written as *rate into* $\mathbf{n}$ = *rate out of* $\mathbf{n}$, is expressed by:

$$\sum_{k=1}^{K} (N_k - n_k + 1) v_k(\mathbf{n}_k^-) P(\mathbf{n}_k^-) + \sum_{k=1}^{K} (n_k + 1) \mu_k P(\mathbf{n}_k^+) = \sum_{k=1}^{K} (N_k - n_k) v_k(\mathbf{n}) P(\mathbf{n}) + \sum_{k=1}^{K} n_k \mu_k P(\mathbf{n}) \quad (16)$$

where:

$$v_k(\mathbf{n}) = \begin{cases} v_k & \text{for } C - \mathbf{nc} > c_k + \lfloor R_{r,k} \rfloor \\ (1-z_k) v_k & \text{for } C - \mathbf{nc} = c_k + \lfloor R_{r,k} \rfloor \\ 0 & \text{otherwise} \end{cases} \quad (17)$$

$\mathbf{c} = (c_1, \dots, c_K)$,
$\mathbf{n}_k^+ = (n_1, \dots, n_{k-1}, n_k + 1, n_{k+1}, \dots, n_K)$,
$\mathbf{n}_k^- = (n_1, \dots, n_{k-1}, n_k - 1, n_{k+1}, \dots, n_K)$,
and $P(\mathbf{n}), P(\mathbf{n}_k^-), P(\mathbf{n}_k^+)$ refer to the probability distributions of states $\mathbf{n}, \mathbf{n}_k^-, \mathbf{n}_k^+$, respectively.

The proposed model does not have a PFS for the determination of the steady state probabilities $P(\mathbf{n})$ due to the fact that local balance can be destroyed between states $\mathbf{n}_k^-$, $\mathbf{n}$ or $\mathbf{n}, \mathbf{n}_k^+$. This means that $P(\mathbf{n})$'s (and consequently all performance measures) can be determined by solving the global balance equations, a realistic task only for small examples of links with small capacity and two or three service-classes.

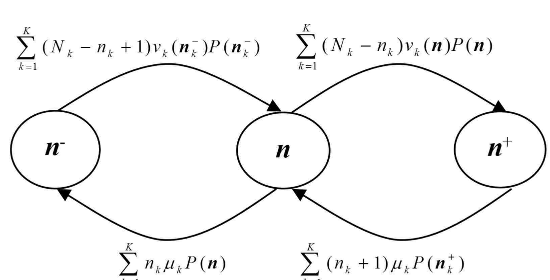

Fig. 2: State transition diagram of the QR-MFCR

To circumvent this problem, an approximate but recursive formula for the calculation of the distribution $G_{\mathrm{f}}(j)$ is proved. By definition:

$$G_{\mathrm{f}}(j) = \sum_{\mathbf{n}\in\mathbf{\Omega}_j} P(\mathbf{n}) \tag{18}$$

where $\mathbf{\Omega}_j = \{\mathbf{n} \in \Omega : \mathbf{nc} = j\}$ and $\mathbf{\Omega} = \{\mathbf{n} : 0 \leq \mathbf{nc} \leq C,\ k = 1, \ldots, K\}$.
Since $j = \mathbf{nc} = \sum_{k=1}^{K} n_k c_k$, (18) is written as:

$$jG_{\mathrm{f}}(j) = \sum_{k=1}^{K} c_k \sum_{\mathbf{n}\in\mathbf{\Omega}_j} n_k P(\mathbf{n}) \tag{19}$$

To determine the $\sum_{\mathbf{n}\in\mathbf{\Omega}_j} n_k P(\mathbf{n})$ in (19), it is assumed that (this is an approximation) local balance exists between the adjacent states $\mathbf{n}_k^-, \mathbf{n}$; this local balance has the following form:

$$(N_k - n_k + 1) l_{k,\mathrm{f}}(\mathbf{n}_k^-) P(\mathbf{n}_k^-) = n_k P(\mathbf{n}) \tag{20}$$

where: $l_{k,\mathrm{f}}(\mathbf{n}_k^-) = v_k(\mathbf{n}_k^-)/\mu_k$.
Summing both sides of (20) over $\mathbf{\Omega}_j$, it results:

$$\sum_{\mathbf{n}\in\mathbf{\Omega}_j} (N_k - n_k + 1) l_{k,\mathrm{f}}(\mathbf{n}_k^-) P(\mathbf{n}_k^-) = \sum_{\mathbf{n}\in\mathbf{\Omega}_j} n_k P(\mathbf{n}) \tag{21}$$

where: $l_{k,\mathrm{f}}(\mathbf{n}_k^-) = v_k(\mathbf{n}_k^-)/\mu_k$.
The left hand side of (21) is written as:

$$\begin{aligned} &\sum_{\mathbf{n}\in\mathbf{\Omega}_j} (N_k - n_k + 1) l_{k,\mathrm{f}}(\mathbf{n}_k^-) P(\mathbf{n}_k^-) = \\ &N_k \sum_{\mathbf{n}\in\mathbf{\Omega}_j} l_{k,\mathrm{f}}(\mathbf{n}_k^-) P(\mathbf{n}_k^-) - \sum_{\mathbf{n}\in\mathbf{\Omega}_j} (n_k - 1) l_{k,\mathrm{f}}(\mathbf{n}_k^-) P(\mathbf{n}_k^-) \end{aligned} \tag{22}$$

Since $\sum_{\mathbf{n}\in\mathbf{\Omega}_j} l_{k,\mathrm{f}}(\mathbf{n}_k^-) P(\mathbf{n}_k^-) = l_{k,\mathrm{f}}(j - c_k) G_{\mathrm{f}}(j - c_k)$, the first term of the right hand side of (22) is written as follows:

$$N_k \sum_{\mathbf{n}\in\mathbf{\Omega}_j} l_{k,\mathrm{f}}(\mathbf{n}_k^-) P(\mathbf{n}_k^-) = N_k l_{k,\mathrm{f}}(j - c_k) G_{\mathrm{f}}(j - c_k) \tag{23}$$

where:

$$l_{k,\mathrm{f}}(j - c_k) = \begin{cases} l_{k,\mathrm{f}} & \text{for } j < C - \lfloor R_{r,k} \rfloor \\ (1 - z_k) l_{k,\mathrm{f}} & \text{for } j = C - \lfloor R_{r,k} \rfloor \\ 0 & \text{otherwise} \end{cases} \tag{24}$$

The second term of the right hand side of (22) becomes:

$$\begin{aligned} &\sum_{\mathbf{n}\in\mathbf{\Omega}_j} (n_k - 1) l_{k,\mathrm{f}}(\mathbf{n}_k^-) P(\mathbf{n}_k^-) = \\ &l_{k,\mathrm{f}}(j - c_k) y_{k,\mathrm{f}}(j - c_k) G_{\mathrm{f}}(j - c_k) \end{aligned} \tag{25}$$

where $y_{k,\mathrm{f}}(j - c_k)$ is the average number of service-class k calls in state $j - c_k$.
Based on (23)-(25), (22) becomes:

$$\begin{aligned} &\sum_{\mathbf{n}\in\mathbf{\Omega}_j} (N_k - n_k + 1) l_{k,\mathrm{f}}(\mathbf{n}_k^-) P(\mathbf{n}_k^-) = \\ &l_{k,\mathrm{f}}(j - c_k)(N_k - y_{k,\mathrm{f}}(j - c_k)) G_{\mathrm{f}}(j - c_k) \end{aligned} \tag{26}$$

Equation (21) due to (26) is written as:

$$(N_k - y_{k,\mathrm{f}}(j - c_k)) l_{k,\mathrm{f}}(j - c_k) G_{\mathrm{f}}(j - c_k) = \sum_{\mathbf{n}\in\mathbf{\Omega}_j} n_k P(\mathbf{n}) \tag{27}$$

Equation (19) due to (27) is written as:

$$jG_{\mathrm{f}}(j) = \sum_{k=1}^{K} (N_k - y_{k,\mathrm{f}}(j - c_k)) l_{k,\mathrm{f}}(j - c_k) c_k G_{\mathrm{f}}(j - c_k) \tag{28}$$

In (28), the values of $y_{k,\mathrm{f}}(j - c_k)$ are unknown. To determine them, the following lemma is considered [33]: two stochastic systems are equivalent and provide the same CBP, if they have: a) the same parameters $(K, N_k, l_{k,\mathrm{f}})$ and b) the same number of states. The purpose is therefore to determine a stochastic system, in order to calculate $y_{k,\mathrm{f}}(j - c_k)$. The channel requirements of calls and the capacity of the new system should meet two criteria: 1) conditions (a) and (b) are valid and 2) each state has a unique value of j.

Now, state j is reached only via $j - c_k$ and therefore $y_{k,\mathrm{f}}(j - c_k) = n_k - 1$. Based on the above, (28) is given by:

$$G_{\mathrm{f}}(j) = \begin{cases} 1 & \text{for } j = 0 \\ \frac{1}{j} \sum_{k=1}^{K} (N_k - n_k + 1) l_{k,\mathrm{f}}(j - c_k) c_k G_{\mathrm{f}}(j - c_k), & \\ & \text{for } j = 1, \ldots, C \\ 0 & \text{otherwise} \end{cases} \tag{29}$$

where $l_{k,\mathrm{f}}(j - c_k)$ is given by (24).

The calculation of $G(j)$'s via (29) requires the unknown value of n_k. These values can be obtained if one determines the state space of the equivalent system. This procedure, however, is quite complex especially for systems of large capacity and many service-classes. Thus, n_k in state j, $n_k(j)$, is approximated by $y_k(j)$, when calls follow a Poisson process. Therefore, $G_{\mathrm{f}}(j)$'s are determined via the formula:

$$G_{\mathrm{f}}(j)=\begin{cases}1 & \text{for } j=0\\ \frac{1}{j}\sum\limits_{k=1}^{K}(N_k-y_k(j-c_k))l_{k,\mathrm{f}}(j-c_k)c_kG_{\mathrm{f}}(j-c_k), & \\ & \text{for } j=1,\ldots,C\\ 0 & \text{otherwise}\end{cases} \quad (30)$$

where the values of $y_k(j)$'s are given by (5).

Having determined $G_{\mathrm{f}}(j)$'s, the calculation of TC probabilities of service-class *k* calls, P_{b_k}, is based on:

$$P_{b_k}=\sum_{j=C-c_k-\lfloor R_{r,k}\rfloor+1}^{C}\frac{G_{\mathrm{f}}(j)}{G_{\mathrm{f}}}+z_k\frac{G_{\mathrm{f}}(C-c_k-\lfloor R_{r,k}\rfloor)}{G_{\mathrm{f}}} \quad (31)$$

where: $G_{\mathrm{f}}=\sum_{j=0}^{C}G_{\mathrm{f}}(j)$.

CC probabilities of service-class k, B_k, can be determined via (31), where G_{f}'s are calculated (via (26)) for a system with N_k-1 traffic sources. Finally, the link utilization can be calculated via (4).

5 Numerical Examples - Evaluation

In this section, we present an application example and provide:

i) analytical CBP results of the proposed MFCR/RTR method and the MFCR model of [5] and

ii) analytical and simulation TC probabilities results of the QR-MFCR model and analytical TC probabilities results of the model of [5].

As a reference, analytical results for Poisson arrivals and the CS policy [18, 28] or the BR policy [29] are presented. Simulation results are derived via Simscript II [30] and are mean values of seven runs.

Consider a link of *C* = 60 channels and *K* = 3 service-classes, with the traffic characteristics of Table 1.

Tab. 1: Traffic characteristics of service-classes

Service -class	**Traffic-load** (erl)	**Bandwidth** (channels)	**MFCR param.** (channels)
1st	$l_1=1.0$	$c_1=1$	$R_{r,1}=9.4$
2nd	$l_2=1.0$	$c_2=5$	$R_{r,2}=5.3$
3rd	$l_4=1.0$	$c_3=10$	$R_{r,3}=0$

In the case of quasi-random traffic, two sets of traffic sources are considered: i) $N_1=N_2=N_3=10$ sources and ii) $N_1=N_2=N_3=30$ sources. In both sets, the values of $l_{k,\mathrm{f}}$ are determined by $l_{k,\mathrm{f}}=l_k/N_k$ for $k=1,2,3$.

The reservation of $R_{r,1} = 9.4$ channels is achieved by considering that with probability 0.4, 10 channels are reserved while, with probability 0.6, 9 channels are reserved. Similarly, the reservation of $R_{r,2} = 5.3$ channels is achieved by reserving 6 and 5 channels with probabilities 0.3 and 0.7, respectively.

Figs. 3-5 are related to the MFCR/RTR method. In their x-axis, the offered traffic load of each service-class increases in steps of $0.5, 0.20$ and 0.1 erl, respectively. So, point 1 is: $(l_1,l_2,l_3)=(1.0,1.0,1.0)$ while point 11 is: $(l_1,l_2,l_3)=(6.0,3.0,2.0)$.

In Figs. 3-5, the analytical CBP results obtained by the MFCR/RTR method, the model of [5] (MFCR policy) and the CS policy together with the MFCR simulation results, for each service-class, respectively, are presented. All three figures show that the MFCR/RTR provides results closer to simulation compared to the CBP results obtained by [5].

A lot of examples reveal that the method of RTR provides closer CBP results compared to the simulation results, especially when CBP tend to be equal among different service-classes. This is intuitively expected, since large MFCR parameters lead to large reservation spaces and therefore to many states *j* whereby $y_k^*(j)$'s should be calculated.

In the x-axis of Figs. 6-8 the offered traffic load of each service-class increases again in steps of 0.5, 0.2 and 0.1 erl, respectively. So, point 8 is: $(l_1,l_2,l_3)=$

$(4.5, 2.4, 1.7)$.

Figs. 6-8 present analytical TC probabilities results of the QR-MFCR, the R-MFCR of [5] and the models of [18, 28, 29] together with the QR-MFCR simulation results, for each service-class, respectively. All figures show that the analytical results of the QR-MFCR model: a) are close to the corresponding simulation results, a fact that validates the proposed formulas, b) are lower than those of the R-MFCR model, especially for $N_1 = N_2 = N_3 = 10$ sources, due to the finite number of traffic sources.

In addition, Figs. 6-8, show that TC probabilities of the 3^{rd} service-class are reduced due to the MFCR policy at the cost of substantially increasing the TC probabilities of the other two service-classes.

6 Conclusion

In this paper, a transition rate method for the CBP calculation in a link that accommodates Poisson arriving calls via the MFCR policy is initially proposed. The proposed method shows that reverse transition rates can appear in the reservation space. The CBP results obtained by the reverse transition method compared to simulation results show that this method improves the accuracy of the analytical calculation. Furthermore, a multirate loss model of quasi-random arriving calls which compete for the available link channels under the MFCR policy is proposed. The analysis of the proposed model leads to approximate formulas for the determination of TC probabilities, CC probabilities and link utilization. These formulas provide quite accurate results compared to simulation.

As a future work, the intention is to extend these models in two directions. In the first one, the applicability of convolution algorithms for the calculation of congestion probabilities in MFCR models will be investigated. Although convolution algorithms are more complicated compared to Kaufman-Roberts based recursive formulas, they have a specific advantage. They preserve micro-state information which is necessary if the admission control is probabilistic or if it requires the knowledge of the number of in-service calls in each state j in order to decide if a new call will be accepted in the system or not. In the second one, different arrival processes will be investigated. A possible arrival process is the batched Poisson arrival process, where calls arrive in the system in batches and the admission control can be applied either to each call separately or to the whole batch. Another arrival process worth to be studied is the ON-OFF arrival process, where new calls alternate between ON (transmission) and OFF (idle) states.

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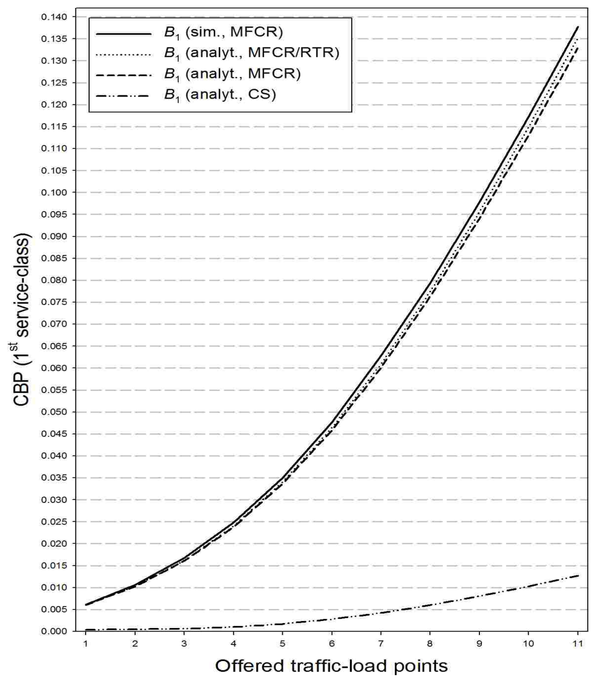

Fig. 3: CBP – 1$^{\mathrm{st}}$ service-class

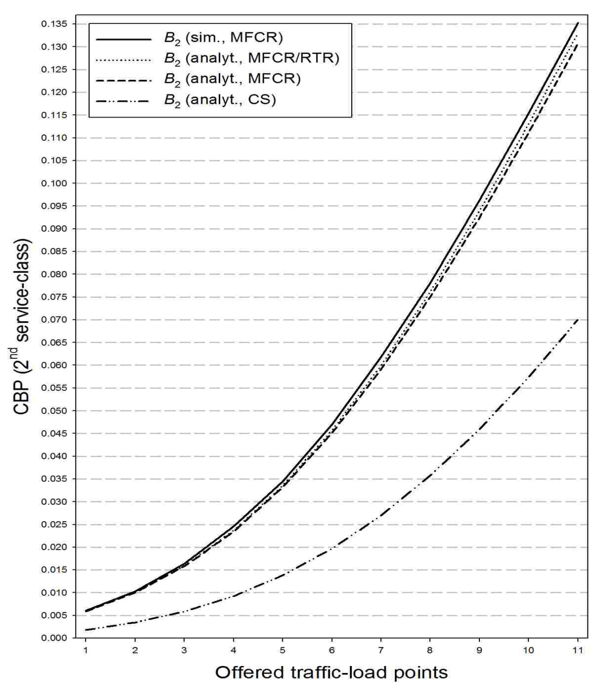

Fig. 4: CBP – 2$^{\mathrm{nd}}$ service-class

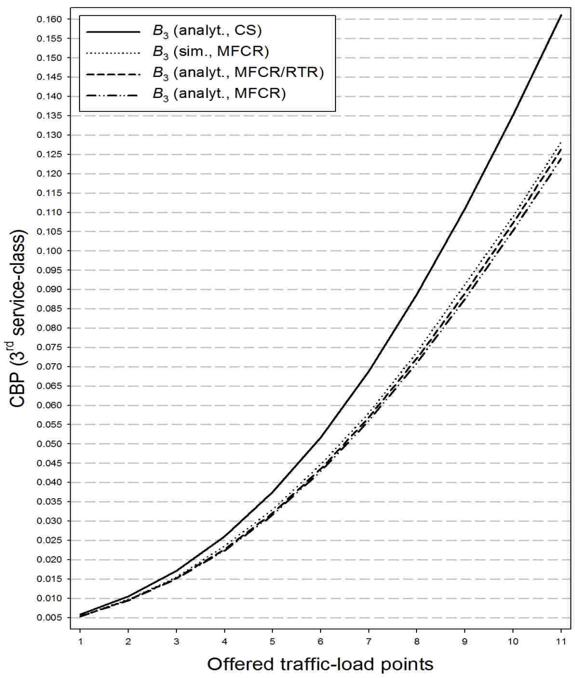

Fig. 5: CBP – 3^{rd} service-class

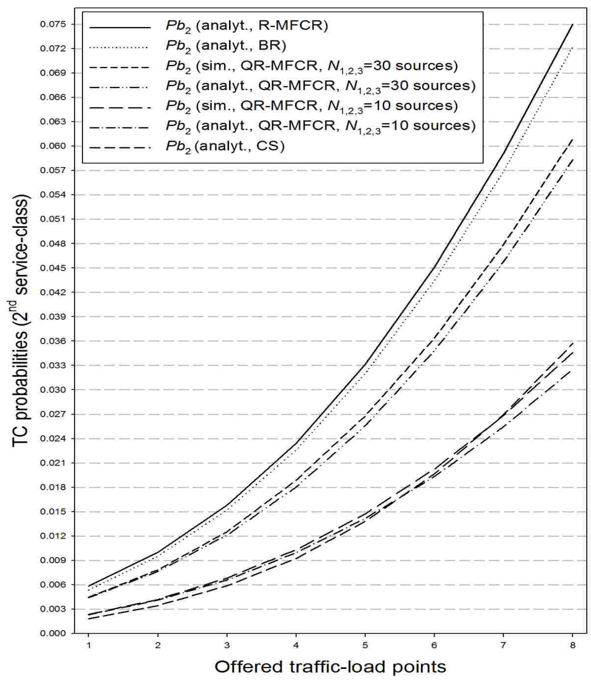

Fig. 7: TC Probabilities – 2^{nd} service-class

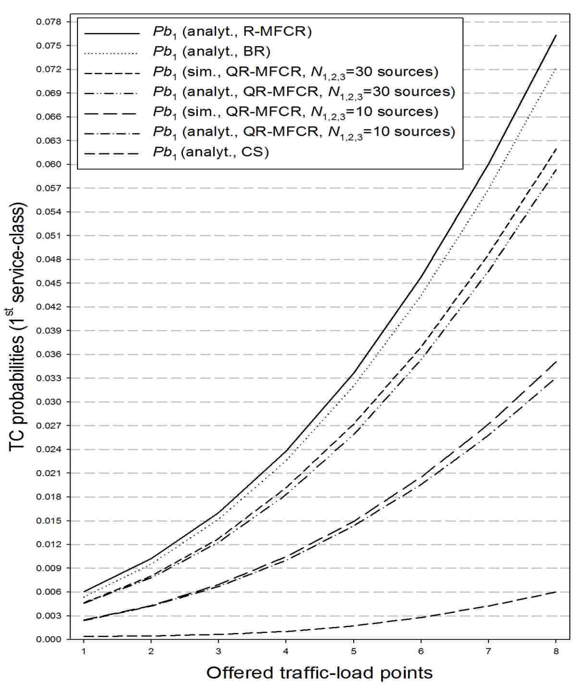

Fig. 6: TC Probabilities – 1^{st} service-class

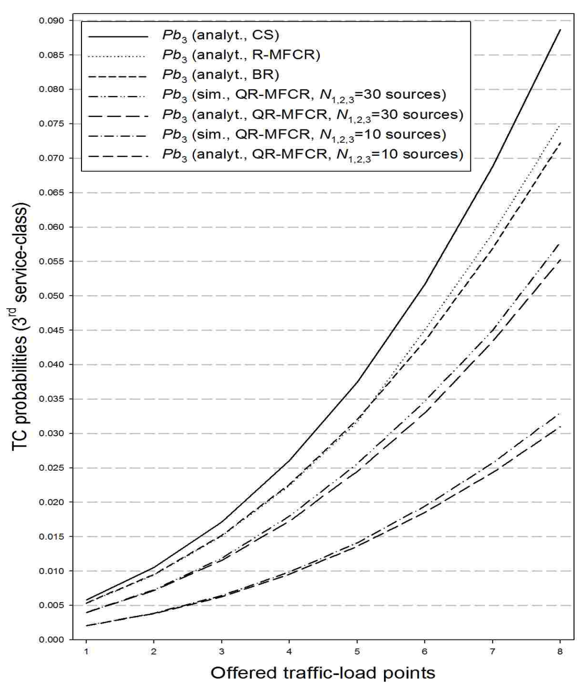

Fig. 8: TC Probabilities – 3^{rd} service-class