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APPLICATION OF THE MOMENT SHAPE REPRESENTATIONS TO THE GENERAL SHAPE ANALYSIS

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Abstract. The General Shape Analysis (GSA) is a task similar to the shape recognition and retrieval. However, in GSA an object usually does not belong to a template class, but can only be similar to some of them. Moreover, the number of applied templates is limited. Usually, ten most general shapes are used. Hence, the GSA consists in searching for the most universal information about them. This is useful when some general information has to be concluded, e.g. in coarse classification. In this paper the result of the application of three shape descriptors based on the moment theory to the GSA is presented. For this purpose the Moment Invariants, Contour Sequence Moments, and Zernike Moments were selected.

Key words. moments, shape representation, General Shape Analysis

1 Introduction

For the General Shape Analysis (GSA) one can apply the same algorithms as for shape recognition and retrieval.

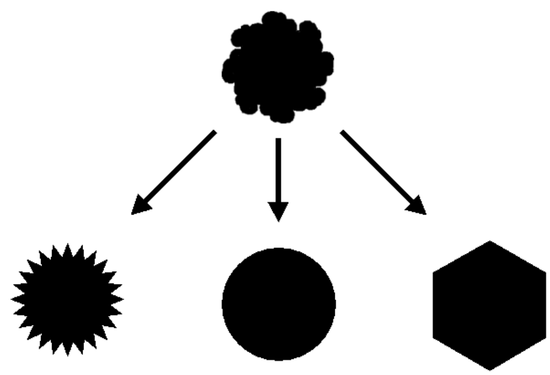

Fig. 1: Pictorial representation of the GSA problem.

However, the GSA is not the same task — here a template is only represented by one object and there are only few base classes. Usually, ten universal shapes are utilized in order to obtain some conclusions (e.g., if the database include the triangle, square, etc., one can conclude to what degree the object is triangular, square, and so on [6]).

The GSA is applied when shape analysis can be considered on a higher level of abstraction. One usage is the initial coarse classification (e.g. in large databases). It is very important for speeding up the classification through first matching the analysed object with a small number of general classes. During subsequent stages the object may be analysed in details within the selected class. This analysis can be repeated in smaller classes. Moreover, the user can control the level of generality.

Given the rapid development of computer vision applications, where the image patterns analysis (ranging from medical diagnosis [1, 10], to analysing materials [4], to artistic applications [11]) requires a significant computation time, any procedure for preclassification can be very valuable.

The GSA was successfully applied so far also in the problem of stamp identification (see [5, 8]).

In this paper GSA is performed based on the template matching by means of three moments shape descriptors. For the analysis of the efficiency 187 persons were asked to fill out an inquiry form. The results were then used as a benchmark for the evaluation of artificial methods results.

The analysis of general properties of objects represented in a shape form was also performed in [12–14]. However, in those cases the particular shape features, such as rectangularity, triangularity or ellipticity, were determined individually.

2 The Algorithms Applied in the General Shape Analysis Problem

The Geometric Moments or Moment Invariants (MI) were proposed by Hu in 1962 [2]. The MI are usually applied to greyscale objects, however, if we assume only two possible values, they can be used as a shape descriptor as well. The definition of the MI provided below is based on three publications [3, 9, 15]. General geometrical moments are derived using the equation (in a discrete version):

$$M_{pq} = \int_{-\infty}^{\infty} x^p y^q f(x,y) \mathrm{d}x \mathrm{d}y, \tag{1}$$

where: $p, q = 0, 1, \ldots, \infty$. The above equation can be written in a discrete version:

$$m_{pq} = \sum_x \sum_y x^p y^q f(x,y), \tag{2}$$

where: $p, q = 0, 1, \ldots, \infty$. In case of shape objects $f(x,y)$ can be equal to 1 if a pixel belongs to the object, and 0 otherwise.

The centroid is obtained by means of:

$$x_c = \frac{m_{10}}{m_{00}}, \qquad y_c = \frac{m_{01}}{m_{00}}, \tag{3}$$

Later, the following are obtained:

$$\mu_{pq} = \sum_x \sum_y (x - x_c)^p (y - y_c)^q f(x,y), \tag{4}$$

and central normalised moments are expressed as:

$$\eta_{pq} = \frac{\mu_{pq}}{\mu_{00}^{\frac{p+q+2}{2}}}. \tag{5}$$

Finally, the MI are obtained. Usually, the first seven values are applied.

The second shape descriptor is the Contour Sequence Moments (CSM), which works only on the outer contour of a shape. The below description is based on [16].

The one-dimensional normalised CSM are given by:

$$m_r = \frac{1}{N}\sum_{i=1}^{N}[z(i)]^r, \qquad \mu_r = \frac{1}{N}\sum_{i=1}^{N}[z(i)-m_1]^r, \tag{6}$$

where:
$z(i)$ contains the distances from the centroid to N elements of a shape.

The r-th normalised CSM and normalised central CSM are represented using the following formulas:

$$\overline{m_r} = \frac{m_r}{(\mu_2)^{\frac{r}{2}}}, \qquad \overline{\mu_r} = \frac{\mu_r}{(\mu_r)^{\frac{r}{2}}}. \tag{7}$$

Finally, the shape representation is obtained:

$$F_1 = \frac{(\mu_2)^{\frac{1}{2}}}{m_1}, F_2 = \frac{\mu_3}{(\mu_2)^{\frac{3}{2}}}, F_3 = \frac{\mu_4}{(\mu_2)^2}, F_4 = \overline{\mu_5}. \tag{8}$$

The third shape descriptor is the Zernike Moments (ZM). The following description is based on [7, 17]. For an image $f(x, y)$, 2-D complex ZM are represented as:

$$Z_{pq} = \frac{p+1}{\pi} \iint\limits_{x^2+y^2 \leq 1} V_{pq}^*(x, y) f(x, y) \mathrm{d}x \mathrm{d}y, \quad (9)$$

with the orthogonality relation:

$$\iint\limits_{x^2+y^2 \leq 1} V_{pq}^*(x, y) V_{p'q'}(x, y) \mathrm{d}x \mathrm{d}y = \frac{\pi}{p+1} \delta_{pp'} \delta_{qq'}, \quad (10)$$

where:
p , q — the order and repetition of ZM,
* is the complex conjugation.

The size of the image should be changed in order to have the unit square $S = \{(x, y) \in R^2 : -1 \leq x, y \leq 1\}$. Next, a round subpart is selected from the square S, within the unit disc $D = \{(x, y) \in R^2 : x^2 + y^2 \leq 1\}$, $D \subset S$.

For p and q and $k = q, \ldots, p$, $s = \frac{p-k}{2}$, Zernike polynomials of order p are written as:

$$V_{pq}(x, y) = \sum_{k=q}^{p} B_{pqk} r^k e^{jq\theta}, \quad (11)$$

where B_{pqk} is calculated as follows:

$$B_{pqk} = \frac{(-1)^{\frac{p-k}{2}} (\frac{p+k}{2})!}{(\frac{p-k}{2})!(\frac{k+q}{2})!(\frac{k-q}{2})!}. \quad (12)$$

Given the above definitions, the polar representation is used for the ZM:

$$Z_{pq} = \frac{p+1}{\pi} \int_0^1 \int_{-\pi}^{\pi} \sum_{k=q}^{p} B_{pqk} r^k e^{-jq\theta} f(r, \theta) r \mathrm{d}r \mathrm{d}\theta, \quad (13)$$

with:
$\mathrm{d}x\mathrm{d}y = r\mathrm{d}r\mathrm{d}\theta$,
$-\pi \leq \theta \leq \pi$.

3 Methodology and Results of the Tests

Three discussed moment shape descriptors were applied in the GSA with the use of 50 shapes, divided into ten general shapes (templates) and forty test objects, and stored as bitmaps 200×200 pixels in size, as it is depicted in Fig. 2.

In each case, test objects and general shapes were de-

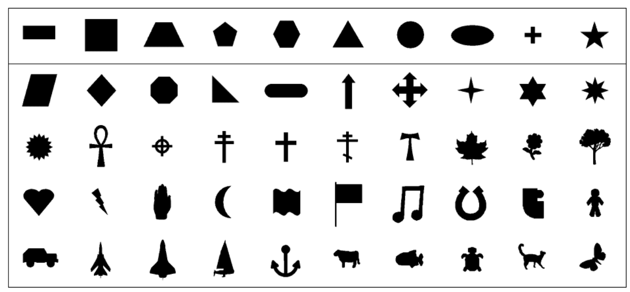

Fig. 2: The 10 template (first row) and 40 test shapes.

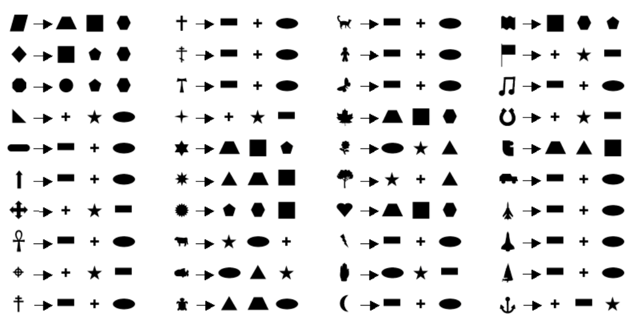

Fig. 3: The test results for the Moment Invariants.

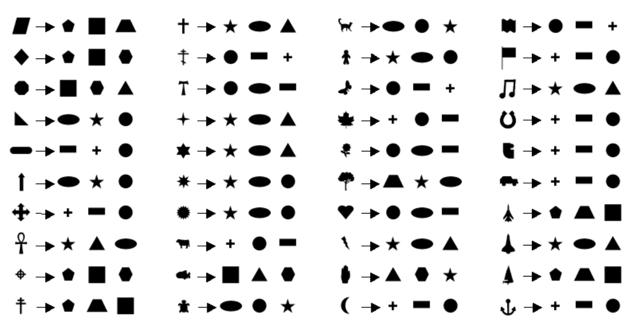

Fig. 4: The test results for the Contour Sequence Moments.

scribed by means of particular shape representation methods. The dissimilarity measure (the Euclidean distance) was calculated between test and template objects. In result, three general shapes most similar to a tested one were selected.

Sample results, obtained for the ZM descriptor, are presented in Fig. 5. However, the graphical form of the results does not allow for an objective judgement of the descriptors' effectiveness. Hence, as a benchmark, an inquiry form was prepared and filled out by 187 people. The evaluation was performed as the comparison between a test result and this benchmark (see Tab. 1). As we can see, the ZM significantly outperform the remaining two techniques.

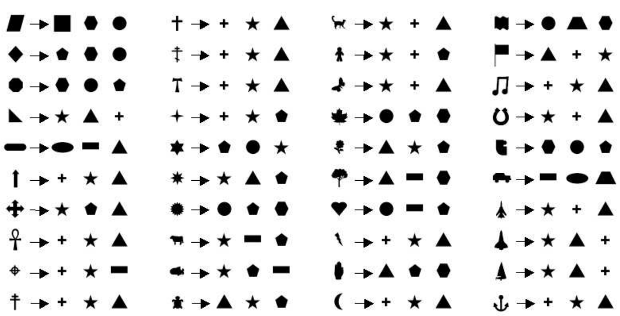

Fig. 5: The test results for the Zernike Moments.

Tab. 1: A comparison of the results obtained by the shape descriptors and humans: the percentage of convergence between a shape descriptor and benchmark result.

Shape descriptor	1st indication	2nd indication	3rd indication
Moment Invariants	20%	17.5%	5%
Contour Sequence Moments	12.5%	12.5%	10%
Zernike Moments	40%	22.5%	20%

Tab. 2: The convergence between shape descriptors and benchmark results. The indication is considered proper if it matches any of the three benchmark indications.

Shape descriptor	1st indication	2nd indication	3rd indication
Moment Invariants	37.5%	37.5%	27.5%
Contour Sequence Moments	50%	27.5%	37.5%
Zernike Moments	62.5%	47.5%	32.5%

The results in Tab. 1 were based on the convergence with the benchmark, derived separately for each template, maintaining the indications sequence. However, the insight into the inquiry results shows that the indications therein vary significantly. Often the same templates are chosen, but in a different sequence. Hence, a second method of evaluation was used — only the presence of a particular indication in the benchmark set was considered (see Tab. 2). Again, the ZM outperformed the other algorithms. However, the results of other two evaluated methods were different, the CSM being significantly better than the MI.

4 Concluding Remarks

In this paper the results of experimental comparison of three shape descriptors based on the moment theory applied to the problem of General Shape Analysis were provided. In this problem the object is matched with a small number of templates (up to ten) and three to five the most similar templates are indicated. What is important, the analysed shape does not belong to any of the template classes, but is only similar to them. In result, we can conclude some general information about it, e.g. how round, triangular or rectangular it is. The General Shape Analysis can be used in the applications where not the precise information about a shape is needed, but only some general conclusions about it. As it has been already mentioned, this usually leads to the formulation of similarity of an object to some basic shapes.

Amongst the three explored methods the best results were obtained using the Zernike Moments. Their results can be considered as very promising, since even the indications given by humans are often ambiguous. The other investigated shape descriptors — the Moment Invariants and Contour Sequence Moments — gave significantly worse results. The performance of the particular algorithms was measured by means of an inquiry form that was developed specially for this purpose. This was the same as the performed tests and it was filled out by almost two hundred persons. The artificial method with results the most similar to the ones provided by humans was treated as the best one.

References

[1] Bator, M., Chmielewski, L.J. (2009). Finding regions of interest for cancerous masses enhanced by elimination of linear structures and considerations on detection correctness measures in mammography. Pattern Analysis and Applications, 12(4), 377–390

[2] Hu, M.K. (1962). Visual Pattern Recognition by Moment Invariants. IEEE Transactions on Information Theory, 8, 179–187

[3] Hupkens, Th.M., Clippeleir, J. de (1995). Noise and Intensity Invariant Moments. Pattern Recognition Letters, 16 (4), 371–376

[4] Khan, M.S., Coenen, F., Dixon, C., El-Salhi, S. (2012). A Classification Based Approach for Predicting Springback in Sheet Metal Forming. Journal of Theoretical and Applied Computer Science 6(2), 45–59

[5] Forczmanski P., Frejlichowski D. (2010). Robust Stamps Detection and Classification by Means of General Shape Analysis. Lecture Notes in Computer Science, 6374, 360–367

[6] Frejlichowski, D. (2010). An Experimental Comparison of Seven Shape Descriptors in the General Shape Analysis Problem. Lecture Notes in Computer Science, 6111, 294–305

[7] Frejlichowski, D. (2011). The Application of the Zernike Moments to the Problem of General Shape Analysis. Control and Cybernetics, 40(2), 515–526

[8] Frejlichowski, D., Forczmanski, P. (2010). General Shape Analysis Applied to Stamps Retrieval from Scanned Documents. Lecture Notes in Computer Science, 6304, 251–260

[9] Liu, C.-B., Ahuja, N. (2004). Vision Based Fire Detection. Proc. of the 17th Int. Conf. on Pattern Recognition, ICPR 2004, Cambridge, UK

[10] Oszutowska-Mazurek, D., Mazurek, P., Sycz, K., Waker-Wojciuk, G. (2012). Estimation of Fractal Dimension According to Optical Density of Cell Nuclei in Papanicolaou Smears. Lecture Notes in Computer Science, 7339, 456–463

[11] Reverter, F., Rosado, P., Figueras, E., Planas, M.A. (2012). Computer vision methods for image-based artistic ideation. Journal of Theoretical and Applied Computer Science, 6(2), 72–78

[12] Rosin, P.L. (1999). Measuring Rectangularity. Machine Vision and Applications, 11, 191–196

[13] Rosin, P.L. (2003). Measuring Shape: Ellipticity, Rectangularity and Triangularity. Machine Vision and Applications, 14, 172–184

[14] Rosin, P.L. (2005). Computing Global Shape Measures. In: Chen, C.H., Wang, P.S.P. (Eds.) Handbook of Pattern Recognition and Computer Vision, 3rd edn., 177–196

[15] Rothe, I., Susse, H., Voss, K. (1996). The Method of Normalization to Determine Invariants. IEEE Trans. On Pattern Analysis and Machine Intelligence, 18, 366–375

[16] Sonka, M., Hlavac, V., Boyle, R. (1998). Image Processing, Analysis, and Machine Vision The book (2nd Edition)

[17] Wee, C.-Y., Paramesran, R. (2007). On the Computational Aspects of Zernike Moments. Image and Vision Computing, 25 (6), 967–980