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Perturbation Analysis of the LMI-Based Continuous-time Linear Quadratic Regulator Problem for Descriptor Systems

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Key Words: *Continuous-time descriptor systems; perturbation analysis; linear quadratic regulator LMI based control problem.*

Abstract. *This paper considers an approach to perform perturbation analysis of linear quadratic regulator (LQR) control problem for continuous-time descriptor systems. The investigated control problem is based on solving LMIs (Linear Matrix Inequalities) and applying Lyapunov functions. The paper is concerned with obtaining linear perturbation bounds for the continuous-time LQR control problem for descriptor systems. The computed perturbation bounds can be used to study the effect of perturbations in system and controller on feasibility and performance of the considered control problem. A numerical example is also presented in the paper.*

1. Introduction

Linear matrix inequalities techniques are inspiring a growing interest in the control community and are emerging as powerful numerical tools for the analysis and design of control systems [1,2,3,4,5]. Numerous problems in control and systems theory can be formulated in terms of linear matrix inequalities. This fact is hardly surprising, given that LMIs are direct byproducts of Lyapunov based criteria, and that Lyapunov techniques play an essential role in the analysis and control of linear systems [1]. The LQR control problem for descriptor/singular systems [10] is a good illustration of what was discussed above. Descriptor systems have been of interest in the literature since they have many important applications in circuit systems [11], robotics [12] and etc. Many classical results in the usual state-space theory as stability, controllability and observability have been extended to these systems [13,14]. Perturbation analysis of some control problems for singular systems is considered in [8,9].

The aim of this paper is to propose an approach to perform linear perturbation analysis of the LMI based LQR control problem for descriptor systems after introducing a suitable right hand part in the used matrix inequalities.

Throughout the paper following notation is applied: $R^{m\times n}$ – the space of real $m\times n$ matrices; $R^n = R^{n\times 1}$; I_n – the identity $n \times n$ matrix; e_n – the unit $n \times 1$ vector; M^T – the transpose of M; $M^{\perp}$ – the pseudo inverse of M; $\|M\|_2=\sigma_{\max}(M)$ – the spectral norm of M, where $\sigma_{\max}(M)$ is the maximum singular value of M; $vec(M) \in R^{mn}$ – the column-wise vector representation of $M \in R^{m\times n}$; $\Pi_{m,n} \in R^{mn\times mn}$ – the vec-permutation matrix, such that $vec(M^T) = \Pi_{m,n} vec(M)$; $M \otimes P$ – the Kroneker product of the matrices M and P. The notation ":=" stands for "equal by definition".

The remaining part of the paper is organized as follows. In Section 2 we propose the problem set up and objective. In Section 3 we describe the performed linear perturbation analysis of the LMI-based continuous-time LQR control problems for singular systems. In Section 4 we present a numerical example, the obtained results and discussions. The paper concludes in Section 5 with some final remarks.

2. Problem Set Up and Objective

Linear continuous-time descriptor systems are generally described by the following set of differential-algebraic equations

$$E\dot{x}(t) = Ax(t) + Bu(t), \quad x(t_0) = x_0 \tag{1}$$

where $x(t) \in R^n$, $u(t) \in R^m$ and $x(t_0) \in R^n$ are the system descriptor state, input and initial conditions, and A, B and E are constant matrices of compatible size.

Definition 2.1. (System equivalence). *Two systems* (E, A, B) *and* $(\hat{E}, \hat{A}, \hat{B})$ *are said to be (system) equivalent, denoted by* $(E, A, B) \approx (\hat{E}, \hat{A}, \hat{B})$, *if there exist nonsingular transformation matrices* $L, R \in R^{n\times n}$ *such that the equations*

$$\hat{E} = LER, \hat{A} = LAR, \hat{B} = LB$$

hold true.

Definition 2.2. (Regularity). *The system is regular, if the polynomial* $\det(sE - A)$ *satisfies* $\det(sE - A) \neq 0$.

Definition 2.3. (Weierstrass normal form). *For any regular system there exist two non-singular matrices* $L, R \in R^{n\times n}$ *such that by*

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = R^{-1}x, x_1 \in R^r, x_2 \in R^{n-r}$$

the following decomposed representation can be obtained

$$\begin{aligned} \dot{x}_1(t) &= \hat{A}_r x_1(t) + \hat{B}_1 u(t) \\ N\dot{x}_2(t) &= x_2(t) + \hat{B}_2 u(t) \end{aligned} \tag{2}$$

Definition 2.4. (Index of nilpotence). *The index of* *nilpotence* ν, *i.e.* $\nu := \min\{q \mid N^q = 0\}$ *is said to be index of a linear descriptor system. Systems with* $\nu \geq 2$ *are called* high *index DAE (differential algebraic systems) systems.*

The descriptor system (1) has a solution for any initial condition and sufficiently smooth input *u*. It is possible that the solution might show impulsive behavior. That is why, consider the system in Weierstrass normal form under sufficiently smooth input, starting from an initial condition x_0. Then the state evolution can be described according to [13]:

$$x_1(t) = e^{\hat{A}_r t} x_{01} + \int_0^t e^{\hat{A}_r (t-\tau)} \hat{B}_1 u(t) d\tau$$

$$x_2(t) = -\sum_{i=1}^{\nu-1} \delta^{(i-1)}(t) N^i x_{02} - \sum_{i=1}^{\nu-1} N^i x_{02} \hat{B}_2 u^{(i)}(t) \qquad (3)$$

Expression (3) for state evolution $x_2(t)$ implies that index one descriptor systems $\nu = 1$ and $N = 0$ will have no impulsive solutions. In this case the system (1) is called impulse free and index one.

Consider the linear continuous-time descriptor system (1), where there is no direct relation between the input and the output signal. Throughout the paper we assume the descriptor system (1) is an index one system.

There exists an equivalent system

$$(\hat{E}, \hat{A}, \hat{B}) = \left(\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} \hat{A}_r & 0 \\ 0 & I_{n-r} \end{bmatrix}, \begin{bmatrix} \hat{B}_1 \\ \hat{B}_2 \end{bmatrix} \right),$$

in Weierstrass canonical form where $A_r \in R^{r \times r}$ is a stable matrix. The transformed system is given as

$$\dot{x}_1(t) = \hat{A}_r x_1(t) + \hat{B}_1 u(t). \qquad (4)$$

The transformed system (4) is obtained using the expression (3b) for state evolution $x_2(t)$.

Linear quadratic regulator problem for descriptor systems [10] means for a given initial state $x(0)$ to find a control law, which minimizes the cost function $\int_0^\infty \left(x_1^T Q_{rp} x_1 + u^T R_p u \right) dt$. It should be found a quadratic Lyapunov function $V(x_1) = x_1^T P_r x_1$, P>0 such that $\frac{d}{dt} V(x_1) < -x_1^T \left[Q_{rp} + K_r^T R_p K_r \right] x_1$. To solve the LQR problem for descriptor systems and to ensure closed-loop stability and specified performance it is necessary to design a state-feedback control $u=K_r x_1$.

In the paper we apply an LMI approach to solve the linear quadratic regulator control problem for singular systems, as stated in [1,10].

$$x_1^T \left[(\hat{A}_r + \hat{B}_1 K_r)^T P_r + P_r (\hat{A}_r + \hat{B}_1 K_r) \right] x_1 < -x_1^T \left(Q_{rp} + K_r^T R_p K_r \right) x_1, \; P_r > 0. \qquad (5)$$

Using Schur complement argument [15] the expression above is equivalent to

$$\begin{bmatrix} (\hat{A}_r + \hat{B}_1 K_r)^T P_r + P_r (\hat{A}_r + \hat{B}_1 K_r) & K_r^T & I \\ K_r & -R_P^{-1} & 0 \\ I & 0 & -Q_{rp}^{-1} \end{bmatrix} < 0, \quad P_r > 0. \qquad (6)$$

We pre- and post-multiply expression (6) by $diag[P_r^{-1}, I, I]$. Further we will use new variables $Q_r = P_r^{-1}$, $Q_r > 0$ and $Y_r = K_r P_r^{-1}$ to obtain the following LMI system:

$$\begin{bmatrix} \hat{A}_r Q_r + Q_r \hat{A}_r^T + \hat{B}_1 Y_r + Y_r^T \hat{B}_1^T & Y_r^T & Q_r \\ Y_r & -R_P^{-1} & 0 \\ Q_r & 0 & -Q_{rp}^{-1} \end{bmatrix} < 0, \quad Q_r > 0. \qquad (7)$$

The main objective of the paper is to conduct a linear sensitivity analysis of the LMI system (7), needed to solve the continuous-time linear quadratic regulator control problem for descriptor systems. Further in the paper we will use the following notation: $R_p^{-1} = R_{ip}$, $Q_{rp}^{-1} = Q_{irp}$, $\Delta R_p^{-1} = \Delta R_{ip}$, $\Delta Q_{rp}^{-1} = \Delta Q_{irp}$.

Throughout the text we assume that the matrices $\hat{A}_r, \hat{B}_1, R_{ip}, Q_{irp}$ are subject to perturbations $\Delta\hat{A}_r, \Delta\hat{B}_1, \Delta R_{ip}, \Delta Q_{irp}$ and assume that they do not change the sign of the LMI system (7). The sensitivity analysis of the continuous-time LMI based linear quadratic regulator problem for singular systems is aimed at determining perturbation bounds of the LMI system (7) as functions of the perturbations in the data $\hat{A}_r, \hat{B}_1, R_{ip}, Q_{irp}$.

3. Linear Perturbation Analysis

In this section perturbation analysis of the LMI (7) for the continuous-time descriptor index one system (4) is performed

$$\begin{bmatrix} \hat{A}_r\hat{B}_1Q_rY_r^{\;T} + \hat{A}_r\hat{B}_1Q_rY_r & (Y_r + \Delta Y_r)^T & (Q_r + \Delta Q_r) \\ (Y_r + \Delta Y_r) & -(R_{ip} + \Delta R_{ip}) & 0 \\ (Q_r + \Delta Q_r) & 0 & -(Q_{irp} + \Delta Q_{irp}) \end{bmatrix} < 0, \tag{8}$$

where

$$\hat{A}_r\hat{B}_1Q_rY_r^{\;T} = (Q_r + \Delta Q_r)(\hat{A}_r + \Delta\hat{A}_r)^T + (Y_r + \Delta Y_r)^T(\hat{B}_1 + \Delta\hat{B}_1)^T$$

$$\hat{A}_r\hat{B}_1Q_rY_r = (Q_r + \Delta Q_r)(\hat{A}_r + \Delta\hat{A}_r) + (Y_r + \Delta Y_r)(\hat{B}_1 + \Delta\hat{B}_1)\cdot$$

It is essential to investigate the effect of the perturbations $\Delta\hat{A}_r, \Delta\hat{B}_1, \Delta R_{ip}, \Delta Q_{irp}$ on the perturbed LMI solutions $Q_r^* + \Delta Q_r$ and $Y_r^* + \Delta Y_r$, where Q_r^*, Y_r^*, and ΔQ_r, ΔY_r, are the nominal solution of the inequality (7) and the perturbations, respectively. The importance of our approach is to perform sensitivity analysis of the inequality (7) similarly to a perturbed matrix equation, after introducing a slightly perturbed suitable right hand part. In this way LMI (9) is obtained

$$\begin{bmatrix} \hat{A}_r\hat{B}_1Q_rY_r^{\;*T} + \hat{A}_r\hat{B}_1Q_rY_r^{\;*} & (Y_r^* + \Delta Y_r^*)^T & (Q_r^* + \Delta Q_r) \\ (Y_r^* + \Delta Y_r^*) & -(R_{ip} + \Delta R_{ip}) & 0 \\ (Q_r^* + \Delta Q_r) & 0 & -(Q_{irp} + \Delta Q_{irp}) \end{bmatrix} = M^* + \Delta M_1 < 0, \tag{9}$$

where

$\hat{A}_r\hat{B}_1Q_rY_r^{\;*T} = (Q_r^* + \Delta Q_r)(\hat{A}_r + \Delta\hat{A}_r)^T + (Y_r^* + \Delta Y_r)^T(\hat{B}_1 + \Delta\hat{B}_1)^T$ and

$\hat{A}_r\hat{B}_1Q_rY_r^* = (Q_r^* + \Delta Q_r)(\hat{A}_r + \Delta\hat{A}_r) + (Y_r^* + \Delta Y_r)(\hat{B}_1 + \Delta\hat{B}_1)$ and M^* is computed using the nominal LMI below

$$\begin{bmatrix} \hat{A}_rQ_r^* + \hat{B}_1Y_r^* + Q_r^*\hat{A}_r^{\;T} + Y_r^{*T}\hat{B}_1^T & Y_r^{*T} & Q_r^* \\ Y_r^* & -R_{ip} & 0 \\ Q_r^* & 0 & -Q_{irp} \end{bmatrix} = M^* < 0, \tag{10}$$

The matrix ΔM_1 includes information regarding data and closed-loop performance perturbations, the rounding errors and the sensitivity of the interior point method that is used to solve the LMIs. Using the relation (10) the perturbed equation (9) can be written in the following way

$$\Delta_{Q_r} + \Omega_{Q_r} = \Delta M_1, \tag{11}$$

where

$$\Delta_{Q_r} = \begin{bmatrix} \hat{A}_r\,\Delta Q_r + \Delta Q_r\,\hat{A}_r^{\;T} & 0 & \Delta Q_r \\ 0 & 0 & 0 \\ \Delta Q_r & 0 & 0 \end{bmatrix},$$

$$\Omega_{Q_r} = \begin{bmatrix} \Delta\hat{A}_rQ_r^* + \hat{B}_1\Delta Y_r + \Delta\hat{B}_1Y_r^* + Q_r^*\Delta\hat{A}_r^{\;T} + \Delta Y_r^T\hat{B}_1^T + Y_r^{*T}\Delta\hat{B}_1^T & \Delta Y_r^T & 0 \\ \Delta Y_r & -\Delta R_{ip} & 0 \\ 0 & 0 & -\Delta Q_{irp} \end{bmatrix}.$$

Due to the fact that we conduct linear perturbation analysis here the terms of second and higher order will be eliminated. Afterwards the vectorized form of expression (11) is expressed below

(12) $vec(\Delta_{Q_r}) + vec(\Omega_{Q_r}) = vec(\Delta M_1),$

where

$$vec(\Delta_{Q_r}) = \left[I \otimes \hat{A}_r + \hat{A}_r \otimes I, 0, I, 0,0,0, I, 0,0 \right]^T vec(\Delta Q_r) := L \Delta q_r,$$

$$vec(\Omega_{Q_r}) =$$

$$= \begin{bmatrix} (Q_r * \otimes I) + (I \otimes Q_r *)\Pi_{n^2} & (I \otimes \hat{B}_1) + (\hat{B}_1 \otimes I)\Pi_{n \times m} & (Y * \otimes I) + (I \otimes Y *^T)\Pi_{m^2} & 0 & 0 \\ 0 & \Pi_{m \times m} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & -I & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -I \end{bmatrix}$$

$$\times \begin{bmatrix} vec(\Delta \hat{A}_r) \\ vec(\Delta Y_r) \\ vec(\Delta \hat{B}_1) \\ vec(\Delta R_{ip}) \\ vec(\Delta Q_{irp}) \end{bmatrix} = \left[L_{t1} \quad L_{t2} \quad L_{t3} \quad L_{t4} \quad L_{t5} \right] \Delta_{a_r\, y_r\, b_1 R\, ipQ\, irp} := L_t \Delta_{a_r\, y_r\, b_1 R\, ipQ\, irp}.$$

The mathematical representations give us the possibility to obtain the relation

(13) $L\Delta q_r + L_{t1} vec(\Delta \hat{A}_r) + L_{t2} vec(\Delta Y_r) + L_{t3} vec(\Delta \hat{B}_1) + L_{t4} vec(\Delta R_{ip}) + L_{t5} vec(\Delta Q_{irp}) = vec(\Delta M_1).$

Finally the relative perturbation bound for the solution Q_r * of the LMI (7) is obtained

(14)
$$\frac{\|\Delta q_r\|_2}{\|vec(Q_r *)\|_2} \le \frac{1}{\|vec(Q_r *)\|_2} \left(L_{a_r y_r b1} \frac{\|vec(\Delta \hat{A}_r)\|_2}{\|vec(\hat{A}_r)\|_2} + L_{a_r y_r b2} \frac{\|vec(\Delta Y_r)\|_2}{\|vec(Y_r *)\|_2} + L_{a_r y_r b3} \frac{\|vec(\Delta \hat{B}_1)\|_2}{\|vec(\hat{B}_1)\|_2} \right) + $$
$$+ \frac{1}{\|vec(Q_r *)\|_2} \left(L_{a_r y_r b4} \frac{\|vec(\Delta R_{ip})\|_2}{\|vec(R_{ip})\|_2} + L_{a_r y_r b5} \frac{\|vec(\Delta Q_{irp})\|_2}{\|vec(Q_{irp})\|_2} + M_1 \frac{\|vec(\Delta M_1)\|_2}{\|vec(M *)\|_2} \right)$$

here

$$\frac{L_{a_r y_r b1}}{\|vec(Q_r^*)\|_2} := \frac{\|L^{\perp}\|_2 \|L_{t1}\|_2 \|vec(\hat{A}_r)\|_2}{\|vec(Q_r^*)\|_2}, \frac{L_{a_r y_r b2}}{\|vec(Q_r^*)\|_2} := \frac{\|L^{\perp}\|_2 \|L_{t2}\|_2 \|vec(Y_r^*)\|_2}{\|vec(Q_r^*)\|_2},$$

$$\frac{L_{a_r y_r b3}}{\|vec(Q_r^*)\|_2} := \frac{\|L^{\perp}\|_2 \|L_{t3}\|_2 \|vec(\hat{B}_1)\|_2}{\|vec(Q_r^*)\|_2}, \frac{M_1}{\|vec(Q_r^*)\|_2} := \frac{\|L^{\perp}\|_2 \|vec(M^*)\|_2}{\|vec(Q_r^*)\|_2},$$

$$\frac{L_{a_r y_r b4}}{\|vec(Q_r^*)\|_2} := \frac{\|L^{\perp}\|_2 \|L_{t4}\|_2 \|vec(R_{ip})\|_2}{\|vec(Q_r^*)\|_2}, \frac{L_{a_r y_r b5}}{\|vec(Q_r^*)\|_2} := \frac{\|L^{\perp}\|_2 \|L_{t5}\|_2 \|vec(Q_{irp})\|_2}{\|vec(Q_r^*)\|_2}$$

are called the individual relative condition numbers of the LMI (7) with respect to the perturbations $\Delta\hat{A}_r$, $\Delta\hat{B}_1$, ΔR_{ip}, ΔQ_{irp} and ΔY_r.

Applying a similar derivation strategy the relative perturbation bounds for the solution Y_r^* of the LMI (7) can be obtained. We use the following expression

$$\Delta_{Y_r} + \Omega_{Y_r} = \Delta M_2 \tag{15}$$

where

$$\Delta_{Y_r} = \begin{bmatrix} \hat{B}_1 \Delta Y_r + \Delta Y_r^T \hat{B}_1^T & \Delta Y_r^T & 0 \\ \Delta Y_r & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\Omega_{Y_r} = \begin{bmatrix} \hat{A}_r \Delta Q_r + \Delta\hat{A}_r Q_r^* + \Delta\hat{B}_1 Y_r^* + \Delta Q_r \hat{A}_r^T + Q_r^* \Delta\hat{A}_r^T + Y_r^{*T} \Delta\hat{B}_1^T & 0 & \Delta Q_r \\ 0 & -\Delta R_{ip} & 0 \\ \Delta Q_r & 0 & -\Delta Q_{irp} \end{bmatrix}.$$

.

Since we perform linear sensitivity analysis here the terms of second and higher order are neglected. Then the vectorized form of relation (15) can be obtained

$$vec(\Delta_{Y_r}) + vec(\Omega_{Y_r}) = vec(\Delta M_2), \tag{16}$$

where

$$vec(\Delta_{Y_r}) = \left[(I \otimes \hat{B}_1) + (\hat{B}_1 \otimes I)\Pi_{n\times m}, \Pi_{m\times m}, 0, I, 0, 0, 0, 0, 0\right]^T vec(\Delta Y_r) := N\Delta y_r,$$

$$vec(\Omega_{Y_r}) =$$

$$= \begin{bmatrix} (Q_r^* \otimes I) + (I \otimes Q_r^*)\Pi_{n^2} & (I \otimes \hat{A}_r) + (\hat{A}_r \otimes I) & (Y_r^* \otimes I) + (I \otimes Y_r^{*T})\Pi_{m^2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -I & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -I \end{bmatrix}$$

$$\times\begin{bmatrix} vec(\Delta\hat{A}_r) \\ vec(\Delta Q_r) \\ vec(\Delta\hat{B}_1) \\ vec(\Delta R_{ip}) \\ vec(\Delta Q_{irp}) \end{bmatrix} = \begin{bmatrix} N_{t1} & N_{t2} & N_{t3} & N_{t4} & N_{t5} \end{bmatrix} \Delta_{a_r q_r b_1 R_{ip} Q_{irp}} := N_t \Delta_{a_r q_r b_1 R_{ip} Q_{irp}}.$$

The derivations made allow us to obtain the expression

(17) $N\Delta y_r + N_{t1}\, vec(\Delta\hat{A}_r) + N_{t2}\, vec(\Delta Q_r) + N_{t3}\, vec(\Delta\hat{B}_1) + N_{t4}\, vec(\Delta R_{ip}) + N_{t5}\, vec(\Delta Q_{irp}) = vec(\Delta M_2).$

At the end the relative perturbation bound for the solution $Y_r{}^*$ of the LMI (7) is obtained

(18)
$$\begin{aligned} \frac{\|\Delta y_r\|_2}{\|vec(Y_r{}^*)\|_2} \le \frac{1}{\|vec(Y_r{}^*)\|_2}\left(N_{a_rq_rb1}\frac{\|vec(\Delta\hat{A}_r)\|_2}{\|vec(\hat{A}_r)\|_2} + N_{a_rq_rb2}\frac{\|vec(\Delta Q_r)\|_2}{\|vec(Q_r{}^*)\|_2} + N_{a_rq_rb3}\frac{\|vec(\Delta\hat{B}_1)\|_2}{\|vec(\hat{B}_1)\|_2}\right) + \\ + \frac{1}{\|vec(Y_r{}^*)\|_2}\left(N_{a_rq_rb4}\frac{\|vec(\Delta R_{ip})\|_2}{\|vec(R_{ip})\|_2} + N_{a_rq_rb5}\frac{\|vec(\Delta Q_{irp})\|_2}{\|vec(Q_{irp})\|_2} + M_2\frac{\|vec(\Delta M_2)\|_2}{\|vec(M^*)\|_2}\right) \end{aligned}$$

where

$$\frac{N_{a_rq_rb1}}{\|vec(Y_r{}^*)\|_2} := \frac{\|N^{\perp}\|_2\|N_{t1}\|_2\|vec(\hat{A}_r)\|_2}{\|vec(Y_r{}^*)\|_2},\ \frac{N_{a_rq_rb2}}{\|vec(Y_r{}^*)\|_2} := \frac{\|N^{\perp}\|_2\|N_{t2}\|_2\|vec(Q_r{}^*)\|_2}{\|vec(Y_r{}^*)\|_2},$$
$$\frac{N_{a_rq_rb3}}{\|vec(Y_r{}^*)\|_2} := \frac{\|N^{\perp}\|_2\|N_{t3}\|_2\|vec(\hat{B}_1)\|_2}{\|vec(Y_r{}^*)\|_2},\ \frac{M_2}{\|vec(Y_r{}^*)\|_2} := \frac{\|N^{\perp}\|_2\|vec(M^*)\|_2}{\|vec(Y_r{}^*)\|_2},$$
$$\frac{N_{a_rq_rb4}}{\|vec(Y_r{}^*)\|_2} := \frac{\|N^{\perp}\|_2\|N_{t4}\|_2\|vec(R_{ip})\|_2}{\|vec(Y_r{}^*)\|_2},\ \frac{N_{a_rq_rb5}}{\|vec(Y_r{}^*)\|_2} := \frac{\|N^{\perp}\|_2\|N_{t5}\|_2\|vec(Q_{irp})\|_2}{\|vec(Y_r{}^*)\|_2}$$

are the individual relative condition numbers of the LMI (7) with respect to the perturbations $\Delta\hat{A}_r$, $\Delta\hat{B}_1$, ΔR_{ip}, ΔQ_{irp} and ΔQ_r.

4. Numerical Example [13]

Consider the continuous-time index one descriptor system (1) given in Weierstrass normal form, i.e.

$$\hat{E} = \begin{bmatrix} 1 & 0 & \vdots & 0 & 0 \\ 0 & 1 & \vdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \vdots & 0 & 0 \\ 0 & 0 & \vdots & 0 & 0 \end{bmatrix},\ \hat{A} = \begin{bmatrix} -1 & 0 & \vdots & 0 & 0 \\ 0 & -1 & \vdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \vdots & 0 & 0 \\ 0 & 0 & \vdots & 0 & 0 \end{bmatrix},\ \hat{B} = \begin{bmatrix} 1 \\ 1 \\ \cdots \\ 1 \\ 0 \end{bmatrix}.$$

Due to the fact that we would like to obtain linear bounds, perturbations in the system matrices are chosen in order to eliminate second and higher order terms in the derivation procedure, i.e.

$$\Delta \hat{A}_r = A_r \times 10^{-i}, \Delta \hat{B}_1 = \hat{B}_1 \times 10^{-i},$$
$$\Delta R_{ip} = R_{ip} \times 10^{-i}, \Delta Q_{irp} = Q_{irp} \times 10^{-i},$$
$$\Delta M_1 = M_1 * \times 10^{-i}, \Delta M_2 = M_2 * \times 10^{-i},$$
$$\Delta Q_r = Q_r * \times 10^{-i}, \Delta Y_r = Y_r * \times 10^{-i} \text{ for } i = 8,7...4.$$

The perturbed solutions $Q_r * + \Delta Q_r$ and $Y_r * + \Delta Y_r$ are computed applying the method presented in [7] and using the software [4]. Performing the proposed approach the linear relative perturbation bounds for the solutions $Q_r *$ and $Y_r *$ of the LMI system (7) are calculated using expressions (14) and (18), respectively.

The results obtained for different size of perturbations are presented in *the tables.*

Perturbation bounds

	$\frac{\|\Delta q_r\|_2}{\|vec(Q_r *)\|_2}$	Bound(14)	$\frac{\|\Delta y_r\|_2}{\|vec(Y_r *)\|_2}$	Bound(18)
8	$7.2387*10^{-8}$	$1.1425*10^{-7}$	$6.4792*10^{-8}$	$1.0876*10^{-7}$
7	$7.2387*10^{-7}$	$1.1425*10^{-6}$	$6.4792*10^{-7}$	$1.0876*10^{-6}$
6	$7.2387*10^{-6}$	$1.1425*10^{-5}$	$6.4792*10^{-6}$	$1.0876*10^{-5}$
5	$7.2387*10^{-5}$	$1.1425*10^{-4}$	$6.4792*10^{-5}$	$1.0876*10^{-4}$
4	$7.2387*10^{-4}$	$1.1425*10^{-3}$	$6.4792*10^{-4}$	$1.0876*10^{-3}$

Based on the suggested solution method to perform perturbation analysis the continuous-time LMI based linear quadratic regulator control problem for descriptor systems we obtain the perturbation bounds (14) and (18). These bounds are close to the real relative perturbation bounds $\frac{\|\Delta q_r\|_2}{\|vec(Q_r *)\|_2}$ and $\frac{\|\Delta y_r\|_2}{\|vec(Y_r *)\|_2}$, which means that they are good in sense that they are tight.

5. Conclusion

We proposed an approach to compute the linear perturbation bounds of the continuous-time LMI based linear quadratic regulator control problem for descriptor systems. We also suggested how to calculate the estimates of the individual condition numbers for the considered LMIs. Tight linear perturbation bounds were obtained for the matrix inequalities determining the problem solution. The computed perturbation bounds can be used to analyze the feasibility and performance of the considered control problem in presence of perturbations in the system and the controller. Having in mind the obtained theoretical results we have presented a numerical example to vividly express the applicability and performance of the proposed solution approach to investigate the sensitivity of the LMI based linear quadratic regulator control problem for singular systems.

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