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ANALYSIS OF MIXED MODE I/II/III FRACTURE IN FOAM CORE SANDWICH STRUCTURES USING IMPOSED DISPLACEMENT SPLIT CANTILEVER BEAMS

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Abstract. Static fracture in foam core sandwich structures under mixed mode I/II/III loading conditions was studied theoretically. In order to generate such loading conditions, a thread guide was used to impose in-plane displacements of the lower crack arm of a sandwich Split Cantilever Beam (SCB). The upper crack arm was loaded by a transverse force. A three-dimensional finite element model of the imposed displacement sandwich SCB configuration was developed. The fracture was studied applying the concepts of linear-elastic fracture mechanics. The strain energy release rate mode components distribution along the crack front was analyzed using the virtual crack closure technique. The influence of the imposed displacement magnitude and the crack length on the fracture was evaluated. The effect of the sandwich core material on the mixed-mode I/II/III fracture was studied. For this purpose, finite element simulations were carried-out assuming that the core is made by different rigid cellular foams. It was found that the strain energy release rate decreases when the foam density increases.

Key words: Mixed mode I/II/III crack, Strain energy release rate, Foam core sandwich structure, Linear-elastic fracture mechanics.

1. Introduction

Foam core sandwich structures find increasing use in load-bearing applications in automobile industry, shipbuilding, aerospace, and civil engineering, due to their high specific stiffness and strength foam. In these applications, the sandwich structures are subjected often to combined loads. Thus, there is an increasing need for knowledge about the mixed mode I/II/III fracture behaviour of these structures. The cracks in the sandwich core are especially insidious, because they may easily escape detection. At the same time, these cracks may cause a dramatic loss of stiffness and compressive strength. Therefore, fracture in the sandwich structures is a subject of intensive research for many years.

Numerous works on sandwich structures fracture and their constituents have been reported in the literature [1–11]. They include both analytical and numerical studies. Linear-elastic fracture mechanics principles and tools together with finite element modelling have been frequently employed in the fracture analysis. A great deal of efforts has been expended in characterizing mode I and mode II fracture behaviour of foam core sandwiches. For this purpose, the Double Cantilever Beam (DCB) and the End Notch Flexure (ENF) configurations have been used. Well developed analytical and numerical methods exist for analyzing the fracture in these configurations.

However, mixed mode I/II/III fracture behaviour of foam core sandwich structures has been relatively less investigated in the specialized literature. This is in contrast with the fact that the inter-laminar fracture in composite laminates under mixed mode I/II/III crack loading conditions has been investigated recently by Szekrenyes by using pre-stressed beam specimens [12]. The mode I component of the loading is obtained by inserting a steel roller between the crack faces. A constant specimen end transverse displacement is induced by a set screw in order to obtain the mode II loading component. The mode III crack loading is provided by the testing machine [12]. Pre-stressed composite beam specimens have also been successfully used to investigate delamination, under mixed-mode II/III crack loading conditions [13, 14].

The aim of the present paper is to analyze theoretically the fracture in foam core sandwich beams under mixed mode I/II/III crack loading conditions. In order to generate such loading conditions, it was proposed here to impose in-plane displacements on the lower crack arm of a sandwich SCB, while the upper arm was loaded by a transverse force. Three-dimensional finite element analysis was carried-out on the foam core sandwich SCB configuration to determine the strain energy release rate mode components for different magnitude of the imposed displacement. For this purpose, linear-elastic fracture mechanics was applied. The influence of the crack length on the mixed mode I/II/III fracture behaviour was evaluated. The effect of the foam core on the fracture was studied too.

2. Numerical estimates

The imposed foam core sandwich SCB configuration considered in the present paper is reported schematically in Fig. 1. The sandwich consists of

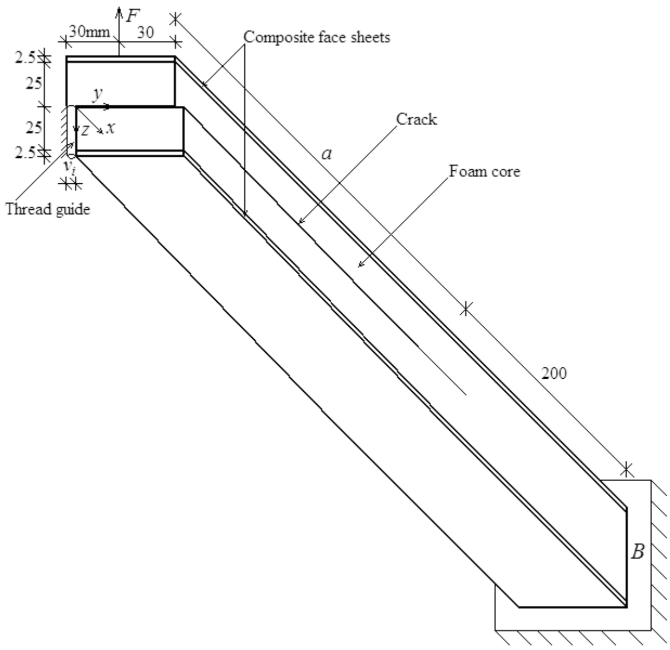

Fig. 1. An overview of the foam core sandwich SCB configuration with imposed in-plane displacement v_i on the lower crack arm

two composite face sheets with thickness of 2.5 mm, each adhesively bonded to a light-weight foam core with thickness of 50 mm. The overall dimensions of the sandwich SCB configuration are 55 mm × 60 mm × 500 mm. The SCB is clamped in section B. There is a longitudinal crack, a, in the mid-plane of the sandwich beam.

It was proposed here to pre-stress the lower crack arm by imposed in-plane displacement v_i, along the y-axis as illustrated in Fig. 1. For this purpose, a thread guide was used. The imposed displacement magnitude can be regulated by locking the thread guide in certain position. The crack arm deformation induced by the imposed in-plane displacement produces mixed mode II/III loading. At the same time, a transverse load F, was applied on the upper crack arm in order to generate mixed mode I/II loading (Fig. 1). It was expected, that the combination between the imposed displacement and the transverse load will generate mixed mode I/II/III crack loading conditions in the sandwich SCB.

The fracture behaviour was studied in terms of the strain energy release rate. The computation of the strain energy release rate mode components

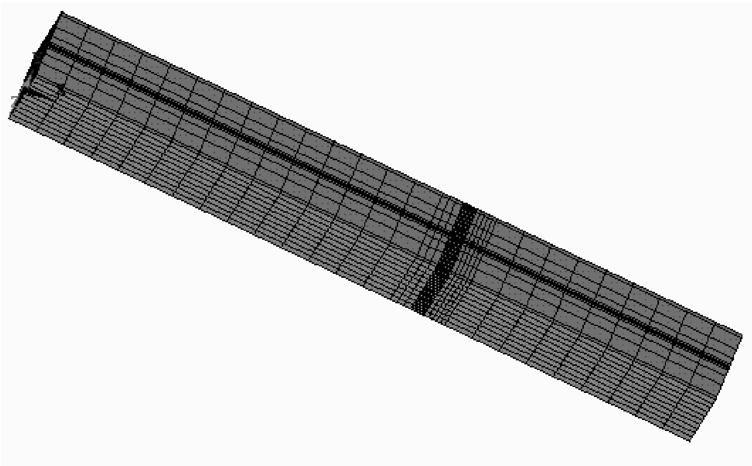

Fig. 2. Finte element discretization of the sandwich SCB

was performed in conjunction with finite element analysis of the pre-stressed sandwich SCB configuration. For this purpose, a three-dimensional linear-elastic finite element model of the foam core composite sandwich SCB was developed using the ANSYS program system. The geometry and dimensions of the model corresponded to these of the sandwich SCB configuration, shown in Fig. 1. Three-dimensional continuum brick finite element SOLID45 was used to mesh the model. This element is defined by 8 nodes (one at each vertex) having three degrees of freedom per node (translation in the nodal x, y, and z directions). A total of 4920 elements were used. The mesh was refined in the crack front region in order to obtain a more exact picture of the strain energy release rate mode components distribution. Before carrying-out further simulations, a mesh sensitivity study was performed, with respect to the elements number in order to ensure that the mesh was fine enough to give reliable results. The mesh used in the finite element simulations is displayed in Fig. 2.

The lower crack arm of the finite element model was loaded by imposed horizontal displacement v_i, applied on the left lateral edge. The upper crack arm was loaded by a transverse force F. All degrees of freedom, i.e. translations in x, y, and z directions of the nodes at the free end of the specimen ($x = 500$ mm), were constrained in order to prevent rigid body movement. The elastic properties of the composite face sheets used in the modelling are reported in Table 1. These properties are defined with respect to the principle material axes x, y, and z, shown in Fig. 1.

Table 1. Elastic properties of the composite face sheets used in the finite element analysis of the sandwich SCB. The subscripts refer to the axes x, y, and z of the coordinate system shown in Fig. 1

E_{xx} (GPa)	E_{yy} (GPa)	E_{zz} (GPa)	G_{xy} (GPa)	G_{yz} (GPa)	G_{zx} (GPa)	ν_{xy}	ν_{yz}	ν_{zx}
45.00	12.00	12.00	4.5	4.5	4.5	0.3	0.3	0.3

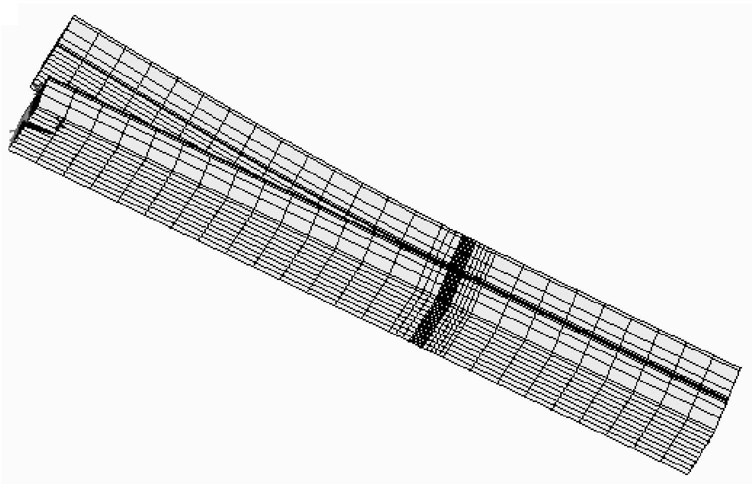

Fig. 3. Deformed shape of the model (the displacements are exaggerated for clarity)

It was assumed that the core is made of different rigid cellular foams (Divinycell H45, Divinycell HT80, and Divinycell HCP100), in order to evaluate the effect of foam core on the mixed-mode I/II/III fracture in the sandwich beam. The elastic properties of these foams are presented in Table 2. The deformed shape of the model is reported in Fig. 3.

The basic purpose of the finite element analysis was to determine the strain energy release rate in the sandwich SCB configuration, at different magnitudes of the imposed displacement v_i, and lengths of the crack, using the virtual crack closure technique. This is a widely used technique, which assumes linear-elastic fracture mechanics validity [15 – 20]. By this technique, the strain energy release rate components, associated with the three basic modes of crack growth, can be calculated separately. The main advantage of this technique is that it uses only one analysis by the finite element model of the sandwich SCB specimen for the actual crack length. The nodal forces at the crack front and the nodal displacements behind the crack front, generated by the finite element model are needed to compute the strain energy release rate mode components.

Table 2. Elastic properties and densities of the rigid cellular foams used in the finite element modelling of the SCB foam core composite sandwich configuration

	E (GPa)	ν	γ (kg/m^3)
Divinycell H45	0.042	0.295	45
Divinycell HT80	0.092	0.420	80
Divinycell HCP100	0.340	0.300	400

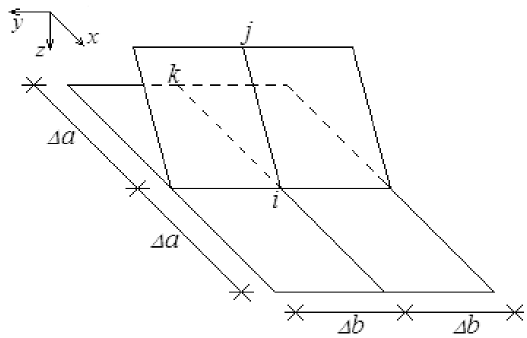

Fig. 4. Schematic of the crack front region with nodal notations

A fragment of the crack front area with nodal notations is depicted in Fig. 4. The virtual crack closure technique assumes, that the work performed to closure the crack by one element is equal to the strain energy released, when the crack grows by one element length. Hence, at node i in Fig. 4, the strain energy release rate mode components G_I, G_{II}, and G_{III}, can be expressed as:

$$G_I = \frac{1}{2\Delta a \Delta b} Z_i \left(w_j - w_k\right), \tag{1}$$

$$G_{II} = \frac{1}{2\Delta a \Delta b} X_i \left(u_j - u_k\right), \tag{2}$$

$$G_{III} = \frac{1}{2\Delta a \Delta b} Y_i \left(v_j - v_k\right), \tag{3}$$

where X, Y, and Z are the nodal force components, and u, v, and w are the nodal displacement components in the x, y, and z directions, respectively. The subscripts in Eqs (1)–(3) refer to the corresponding nodes in Fig. 4. The strain energy release mode components distribution was obtained by applying Eqs (1)–(3) for each node, along the crack front.

Although the virtual crack closure technique is frequently used, because it requires only one analysis by the finite element model, it is an approximate method for the strain energy release rate calculation. Therefore, the results obtained by Eqs. (1)–(3) were verified, using the crack closure technique, which is a direct method for the strain energy release rate calculation. The use of the crack closure technique requires two analyses by the finite element model of the sandwich SCB specimen. The first analysis, which is performed just prior to crack growth, generates the nodal forces at the crack front. The crack advance is simulated in the second analysis by releasing the relevant crack front nodes. The second analysis provides the corresponding displacements needed to compute the strain energy release rate mode components. The crack closure technique assumes that the nodal forces obtained in the first analysis are the forces needed to close the crack. The work performed during the process of crack closure can be determined by multiplying one half of the nodal forces with the corresponding nodal displacements. The strain energy release rate mode components at node i in the crack front (Fig. 4) can be calculated by equations:

$$G_I = \frac{1}{2\Delta a \Delta b} Z_i \delta w_i, \tag{4}$$

$$G_{II} = \frac{1}{2\Delta a \Delta b} X_i \delta u_i, \tag{5}$$

$$G_{III} = \frac{1}{2\Delta a \Delta b} Y_i \delta v_i, \tag{6}$$

where: δu, δv, and δw are the differences in the nodal displacement components. The comparisons indicated, that the difference between the results obtained by the virtual crack closure technique and the crack closure technique was within 1%, which was an indication for the good accuracy of the strain energy release rate analysis.

The strain energy release rate mode components were calculated at three v_i magnitudes (6 mm, 12 mm, and 18 mm), in order to analyze the influence of the imposed displacement v_i, on the fracture. The force magnitude F, remains constant. It was assumed, that the sandwich core is manufactured of Divinycell HCP100 foam. The normalized strain energy release rate mode components distribution along the crack front generated by the finite element analysis, is reported in Fig. 5 (the strain energy release rate mode components are normalized, using the formula $G_i^N = G_i b/F$, where b is the width of the sandwich beam cross-section, and $i = I$, II, and III). The horizontal axis in Fig. 5 was defined such that $y/b = 0.0$ is at the left lateral edge of the sandwich SCB cross-section. Hence, $y/b = 1.0$ is at the right edge. The finite

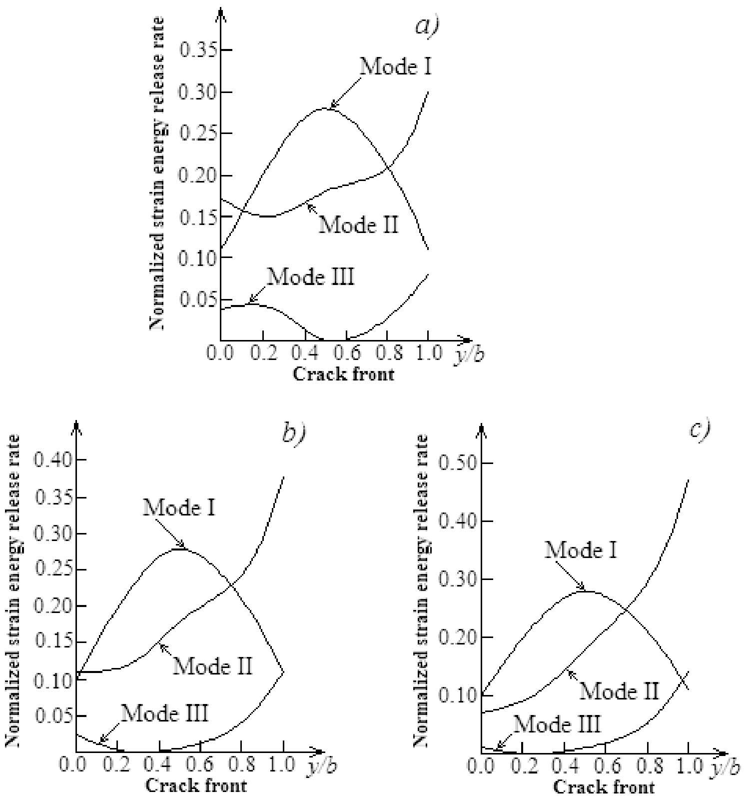

Fig. 5. Normalized strain energy release rate mode components distribution along the crack front in a sandwich SCB with imposed displacement magnitude of: (*a*) 6 mm, (*b*) 12 mm, (*c*) 18 mm. The relative crack length is $a/b = 5$. The horizontal axis is defined such that $y/b = 0.0$ is at the left edge. Thus, $y/b = 1.0$ is at the right edge. Here, b is the sandwich beam cross-section width

element analysis revealed non-uniform distribution of the strain energy release rate mode components along the crack front.

The mode I component of the strain energy release rate has maximum at the center of the crack front and gradually decreases towards the lateral edges (Fig. 5). The mode II component of the strain energy release rate progressively increases towards the right lateral edge. Figure 5 indicates, that the mode III component of the strain energy release rate is low in the central zone of the crack front and increases towards the lateral edges (at the right edge, the mode III component is much higher). This distribution of the mode II and mode III component of the strain energy release rate along the crack front can be attributed to the anti-symmetrical character of the crack arms deformation, induced by the imposed in-plane displacement (Fig. 1).

One can see that the distribution of the mode II and III components of the strain energy release rate changes, when the imposed displacement magnitude increases, while the mode I component is not influenced (Fig. 5). This is due to the fact that the v_i increases, while the transverse load F is constant. The most noticeable change is in the mode II component distribution (at the right edge, the mode II component increases, while at the left edge this component decreases).

The effect of the crack length a, on the mixed mode I/II/III fracture in sandwich SCB with imposed in-plane displacement, was evaluated. Three relative crack lengths (a/b = 1, 3, and 5) were simulated (b is the sandwich beam cross-section width). The imposed displacement was v_i = 18 mm. It was assumed, that the sandwich core is made of Divinycell HCP100 foam. The corresponding strain energy release rate mode components distributions, obtained by the virtual crack closure technique, are depicted in Fig. 6. Since the strain energy release rate distribution along the crack front is non-uniform, the influence of the crack length on the mixed-mode I/II/III fracture behaviour was evaluated by analyzing the mixed-mode ratios of the average values of the strain energy release rate mode components G_I^a/G^a, G_{II}^a/G^a, and G_{III}^a/G^a.

The average values of the strain energy release rate mode components along the crack front, were calculated as:

$$G_I^a = \frac{1}{b}\int_0^b G_I(y)dy, \tag{7}$$

$$G_{II}^a = \frac{1}{b}\int_0^b G_{II}(y)dy, \tag{8}$$

$$G_{III}^a = \frac{1}{b}\int_0^b G_{III}(y)dy, \tag{9}$$

where: $G_I(y)$, $G_{II}(y)$, and $G_{III}(y)$ are the distributions along the crack front calculated by Eqs (1)–(3). The average value of the total strain energy release rate, G^a, along the crack front, was obtained as:

$$G^a = G_I^a + G_{II}^a + G_{III}^a. \tag{10}$$

The mixed-modes of the average values of the strain energy release rate mode components along the crack front, calculated by formulae (7)–(10) (the

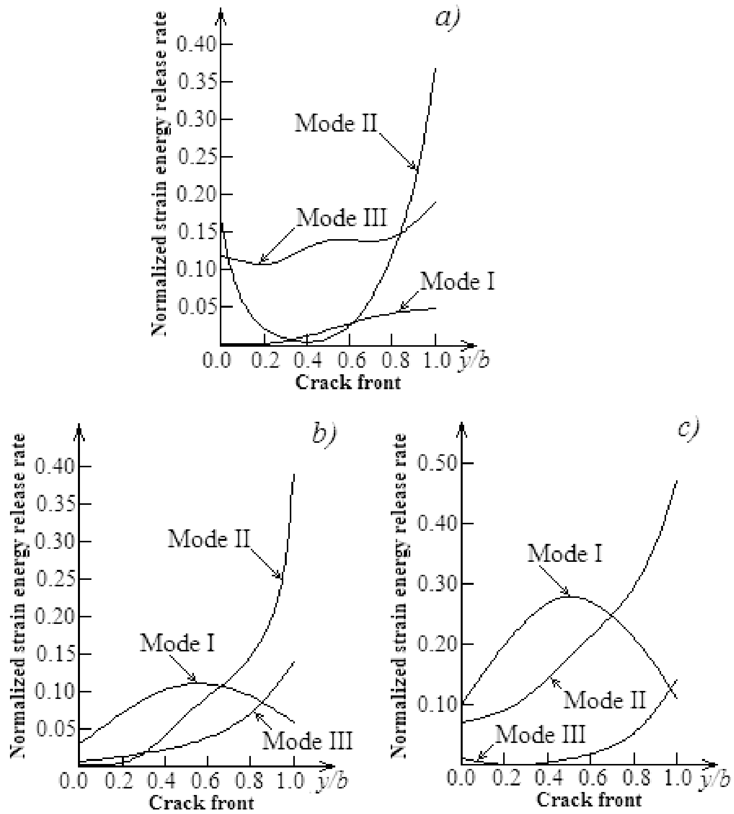

Fig. 6. Normalized strain energy release rate mode components distribution along the crack front in a sandwich SCB with relative crack length $a/b$of: (a) 1, (b) 3, (c) 5. The imposed displacement magnitude is 18 mm. The horizontal axis is defined such that $y/b = 0.0$ is at the left edge. Thus, $y/b = 1.0$ is at the right edge. Here, b is the sandwich beam cross-section width

integrals were solved numerically), at the crack lengths under consideration are reported in Table 3. One can see, that the relative amount of mode I increases, when the crack length increases (this is due to the increase of the bending moment about the y-axis in the upper crack arm, induced by the transverse force, F). The data in Table 3 indicate that the relative amount of mode II increases slightly, when the crack length increases. This finding can be attributed to the fact, that the mode II is induced by the bending of the upper crack arm around the y-axis, induced by the transverse force F and by the bending of the lower crack arm, around the z-axis, induced by the

Table 3. Mixed mode ratios of the average values of the strain energy release rate mode components along the crack front (G_I^a/G^a, G_{II}^a/G^a, and G_{III}^a/G^a) in the foam core composite sandwich SCB at different crack lengths. The imposed displacement magnitude is $v_i = 18$ mm (refer to Fig. 1)

Crack length, a/b	G_I^a/G^a	G_{II}^a/G^a	G_{III}^a/G^a
1	0.077	0.444	0.479
3	0.310	0.485	0.205
5	0.420	0.488	0.092

imposed displacement v_i (please, refer to Fig. 1). The mode II component increases, too, since the bending moment about the y-axis of the upper crack arm increases with increasing the crack length. However, the bending moment about the z-axis of the lower crack arm, induced by imposed displacement decreases with increasing the crack length (the imposed displacement was kept constant $v_i = 18$ mm, at the crack lengths under consideration). As a result of this, the increase of the mode II is lower compared with the increase of the mode I (Table 3). Concerning the mode III, one can see in Table 3, that the relative amount of the mode III decreases with increasing the crack length. This finding can be explained in the following way. The mode III fracture is associated to the shear stress, τ_{xy}, along the crack front (induced by the shear force due to the imposed displacement on the lower crack arm). Since this shear force decreases with increasing the crack length (at one and the same imposed displacement magnitude), the mode III component of the strain energy release rate decreases, too.

One of the basic purposes of the present paper was to evaluate the effect of the sandwich foam the core material on mixed-mode I/II/III fracture. In this relation, finite element simulations were performed, assuming that the sandwich core is made of different rigid cellular foams (Table 2). In these simulations, the relative crack length was $a/b = 5$, the imposed displacement magnitude on the lower crack arm was $v_i = 18$ mm. The strain energy release rate mode components distribution along the crack front in the SCB for the foams under consideration yielded by the finite element analysis is reported in Fig. 7. The diagrams in Fig. 7 clearly indicate, that the strain energy release rate decreases, when the foam elasticity modulus (and the foam density – refer to Table 2) increases. Therefore, the use of rigid cellular foams of higher density for core materials can be recommended as devices for improvement of mixed-mode I/II/III fracture performance of sandwich structures.

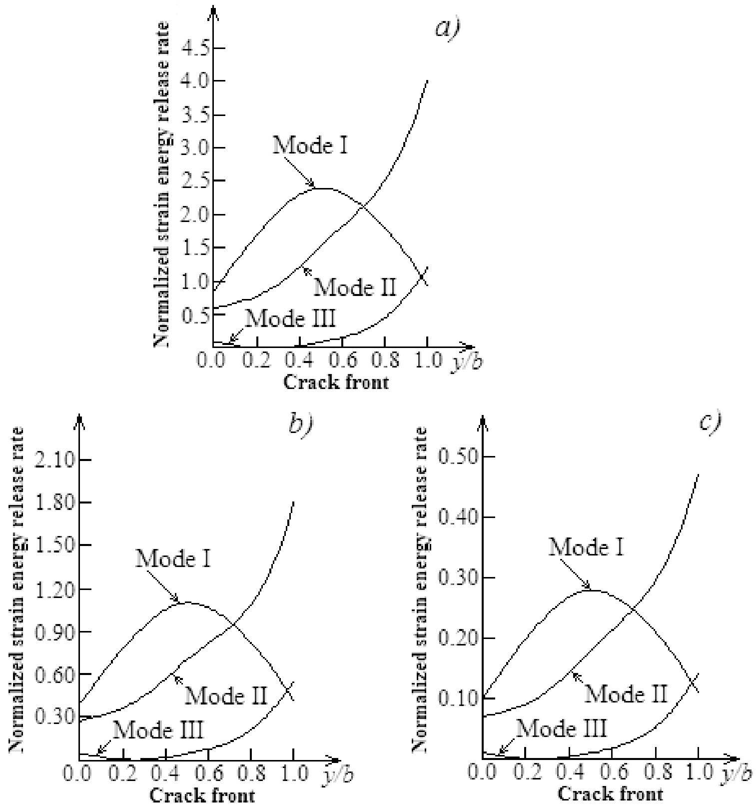

Fig. 7. Normalized strain energy release rate mode components distribution along the crack front in a sandwich SCB with foam core made by (a) – Divinycell H45, (b) – Divinycell HT80, and (c) – Divinycell HCP100. The relative crack length is $a/b = 5$. The imposed displacement magnitude is 18 mm. The horizontal axis is defined such that $y/b = 0.0$ is at the left edge. Thus, $y/b = 1.0$ is at the right edge. Here, b is the sandwich beam cross-section width

3. Conclusions

In the present work, the mixed mode I/II/III fracture in foam core sandwich beams was investigated theoretically by using pre-stressed SCB configuration. The sandwich SCB was pre-stressed by imposed in-plane displacement on the lower crack arm. At the same time, the upper crack arm was loaded by a transverse load. It was expected, that in this way, mixed mode I/II/III crack loading conditions will be induced. The fracture was analyzed, using the concepts of linear-elastic fracture mechanics. For this purpose, a three-dimensional finite element model of the pre-stressed sandwich SCB configuration was developed. The fracture behaviour was studied in terms of the strain energy release rate using the virtual crack closure technique. The simulations yielded non-uniform distribution of the strain energy release rate mode components along the crack front. The effect of the imposed displacement magnitude on the mixed-mode I/II/III fracture was analyzed. The influence of the crack length on the fracture behaviour was evaluated. It was found, that when the imposed displacement and the transverse load are constant, the relative amounts of modes I and II components increase with increasing the crack length. However, the relative amount of mode III component of the strain energy releases rate decreases with increasing the crack length. These findings were attributed to the bending behaviour of the sandwich SCB, induced by the transverse load and the imposed displacement. It can be summarized, that the proposed sandwich SCB configuration with imposed in-plane displacements on the lower crack arm can be used to investigate the mixed mode I/II/III fracture in sandwich structures. The effect of sandwich foam core on the fracture was investigated, too. For this purpose, sandwich configurations with core made of three rigid cellular foams were simulated. The analysis revealed, that strain energy release rate decreases with increase of the foam density. Thus, mixed mode I/II/III fracture performance of sandwich structures can be improved significantly by using foams of higher density for core material.

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