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# SOLID MECHANICS

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## INTERNAL HEAT SOURCE IN THERMOELASTIC MICROELONGATED SOLID UNDER GREEN LINDSAY THEORY\*

PRAVEEN AILAWALIA

*Department of Applied Sciences and Humanities,  
Maharishi Markandeshwar University, Sadopur, Ambala, Haryana, India,  
e-mail: praveen\_2117@rediffmail.com*

SUNIL KUMAR SACHDEVA

*Department of Applied Sciences, D.A.V Institute of Engineering and Technology,  
Jalandhar, Punjab, India  
e-mail: sunilsachdeva.daviet@gmail.com*

and

*Research Scholar, IKG Punjab Technical University,  
Jalandhar, Punjab, India*

DEVINDER SINGH PATHANIA

*Department of Applied Sciences, Guru Nanak Dev Engineering College,  
Ludhiana, Punjab, India*

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**ABSTRACT.** The present study deals with two dimensional deformation, due to internal heat source in a thermoelastic microelongated solid. A mechanical force is applied along the interface of elastic half space and thermoelastic microelongated half space. The problem is in the context of Green Lindsay (GL) theory. The analytic expressions for displacement component, normal force stress, temperature distribution and microelongation have been derived. The effect of internal heat source and microelongation on the derived components have been depicted graphically.

**KEY WORDS:** Thermoelasticity, microelongation, heat source, normal mode, elastic solid.

### 1. Introduction

Classical elasticity is inadequate to discuss the behaviour of materials possessing internal structure. The micropolar elastic model is more realistic than the purely elastic theory to study the response of materials to external stimuli. Eringen and Suhubi [1–2] developed a non linear theory of micro elastic

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\*Corresponding author e-mail: praveen\_2117@rediffmail.com.

solids. Later on, a theory was framed in which material particles in solids can undergo macro-deformations, as well as micro-rotations by Eringen [3–5] and named the theory as “linear theory of micropolar elasticity”.

The axial stretch was included during the rotation of molecules by Eringen [6] and named the theory as micropolar elastic solid with stretch. Nowacki [7], Eringen [8], Tauchert et al. [9], Tauchert [10] and Nowacki and Olszak [11] included thermal effects in the micropolar theory. One can refer to Dhaliwal and Singh [12] for a review on the micropolar thermoelasticity and a historical survey of the subject. The general theory of micromorphic media has been summed up in “Continuum Physics” series by Eringen and Kafadar [13]. Lord and Shulman [14] and coupled theory of elasticity are two important generalized theories of thermoelasticity. Entropy production inequality was proposed by Muller [15]. A generalization of this inequality was proposed by Green and Laws [16]. Green and Lindsay [17] obtained another version of these constitutive equations. These equations were also obtained independently and more explicitly by Suhubi [18]. This theory contains two constants that act as relaxation times and modify all the equations of coupled theory, not only the heat equations. The classical Fourier law of heat conduction is not violated, if the medium under consideration has a centre of symmetry.

Barber [19] studied thermoelastic displacements and stresses due to a heat source moving over the surface of a half plane. Sherief [20] obtained components of stress and temperature distributions in a thermoelastic medium due to a continuous source. Dhaliwal et al. [21] investigated thermoelastic interactions, caused by a continuous line heat source in a homogeneous isotropic unbounded solid. Chandrasekharaiah and Srinath [22] studied thermoelastic interactions due to a continuous point heat source in a homogeneous and isotropic unbounded body. Sharma et al. [23] investigated the disturbance due to a time-harmonic normal point load in a homogeneous isotropic thermoelastic half-space. Sharma and Chauhan [24] discussed mechanical and thermal sources in a generalized thermoelastic half-space. Sarbani and Amitava [25] studied the transient disturbance in a half-space due to moving internal heat source under Lord-Shulman model. Youssef [26] found the solution for the problem on a generalized thermoelastic infinite medium with a spherical cavity, subjected to a moving heat source.

A microelongated elastic solid possesses four degrees of freedom: three for translation and microelongation. In microelongation theory, the material particles can perform only volumetric micro elongation in addition to classical deformation of the medium. The material points of such medium can stretch and contract independently of their translations. Solid liquid crystals,

composite materials reinforced with chopped elastic fibres, porous media with pores filled with non-viscous fluid or gas can be categorized as microelongated medium. Shaw and Mukhopadhyay [27] discussed the variation of periodical heat source response in a functionally graded microelongated medium. Shaw and Mukhopadhyay [28] studied the thermoelastic interactions in a microelongated, isotropic, homogeneous medium in the presence of a moving heat source. Ailawalia and Sachdeva discussed plane strain problem in a thermoelastic microelongated solid with an overlying infinite non-viscous fluid [34].

In the present problem, deformation due to internal heat source in a thermoelastic microelongated solid along the interface of elastic half space has been discussed. The normal mode analysis is applied to derive the expressions for the considered variables for Green Lindsay (GL) theory of thermoelasticity and the variations of the considered variables are represented graphically.

## 2. Problem formulation

The constitutive equation for a homogeneous, isotropic, microelongated, thermoelastic solid are [28]:

$$(1) \quad \sigma_{kl} = \lambda \delta_{kl} u_{r,r} + \mu (u_{k,l} + u_{l,k}) - \beta_0 \left( 1 + t_1 \delta_{2k} \frac{\partial}{\partial t} \right) T \delta_{kl} + \lambda_0 \delta_{kl} \varphi,$$

$$(2) \quad m_k = a_0 \varphi_{,k},$$

$$(3) \quad s - \sigma = \lambda_0 u_{k,k} - \beta_1 \left( 1 + t_1 \delta_{2k} \frac{\partial}{\partial t} \right) T + \lambda_1 \varphi,$$

$$(4) \quad q_k = \frac{K^*}{T_0} T_{,k},$$

where,  $\beta_0 = (3\lambda + 2\mu)\alpha_{t_1}$ ,  $\beta_1 = (3\lambda + 2\mu)\alpha_{t_2}$ .

The field equation of motion, according to [29, 30] and heat conduction equation according to [31] for the displacement, microelongation and temperature changes are:

$$(5) \quad (\lambda + \mu) u_{j,ij} + \mu u_{i,jj} - \beta_0 \left( 1 + t_1 \delta_{2k} \frac{\partial}{\partial t} \right) T_{,i} + \lambda_0 \varphi_{,i} = \rho \ddot{u}_i,$$

$$(6) \quad a_0 \varphi_{,ii} + \beta_1 \left( 1 + t_1 \delta_{2k} \frac{\partial}{\partial t} \right) T - \lambda_1 \varphi - \lambda_0 u_{j,j} = \frac{1}{2} \rho j_0 \ddot{\varphi},$$

$$(7) \quad K^* T_{,ii} - \rho C^* \left( 1 + t_0 \delta_{1k} \frac{\partial}{\partial t} \right) \dot{T} - \beta_0 T_0 \left( 1 + t_0 \delta_{1k} \frac{\partial}{\partial t} \right) \dot{u}_{k,k} - \beta_1 T_0 \dot{\varphi} + \rho \left( 1 + t_0 \delta_{1k} \frac{\partial}{\partial t} \right) Q = 0.$$

The equation of motion and stress components in an elastic medium are given by [32]:

$$(8) \quad (\lambda^e + \mu^e) u_{j,ij}^e + \mu^e u_{,ii}^e = \rho^e \ddot{u}_i^e,$$

$$(9) \quad \sigma_{ij}^e = \lambda^e u_{j,j}^e \delta_{ij} + \mu^e (u_{i,j}^e + u_{j,i}^e).$$

We consider a normal force of magnitude  $P_1$ , acting along the interface of microelongated thermoelastic solid half space, occupying the region  $0 \leq x \leq \infty$  and an elastic solid half space in the region  $-\infty \leq x \leq 0$ , as shown in Fig. 1. We have restricted our analysis to the plane strain parallel to  $xy$  plane with displacement vector:

$$(9a) \quad \vec{u}_i = (u_1, u_2, 0) \quad \text{and} \quad \vec{u}_i^e = (u_1^e, u_2^e, 0).$$

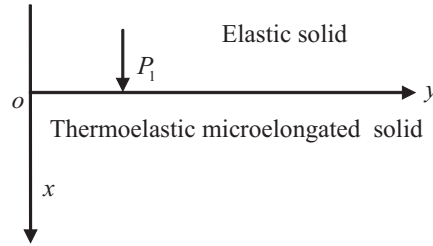


Fig. 1. Geometry of the problem

To simplify calculations, we use following non-dimensional variables:

$$x' = \frac{\omega^*}{c_1} x, \quad y' = \frac{\omega^*}{c_1} y, \quad u_i' = \frac{\omega^* \rho c_1}{\beta_0 T_0} u_i, \quad u_i^{e'} = \frac{\omega^* \rho c_1}{\beta_0 T_0} u_i^e, \quad t' = \omega^* t,$$

$$t'_0 = \omega^* t_0, \quad t'_1 = \omega^* t_1, \quad \sigma'_{ij} = \frac{\sigma_{ij}}{\beta_0 T_0}, \quad \varphi' = \frac{\lambda_0}{\beta_0 T_0} \varphi, \quad \sigma_{ij}^{e'} = \frac{\sigma_{ij}^e}{\beta_0 T_0},$$

$$P'_1 = \frac{P_1}{\beta_0 T_0}, \quad T' = \frac{T}{T_0}, \quad Q' = \frac{1}{\omega^* c_1^2} Q,$$

where,  $\omega^* = \frac{\rho c_1^2 C^*}{K^*}$ ,  $c_1^2 = \frac{\lambda + 2\mu}{\rho}$ .

Using above non dimensional variables and (9a) in equations (5)–(7), we obtain the following non dimensional equations (after dropping superscripts):

$$(10) \quad l_1 \frac{\partial^2 u_1}{\partial x^2} + l_2 \frac{\partial^2 u_2}{\partial x \partial y} + l_3 \frac{\partial^2 u_1}{\partial y^2} - \left(1 + t_1 \delta_{2k} \frac{\partial}{\partial t}\right) \frac{\partial T}{\partial x} + \frac{\partial \varphi}{\partial x} = \frac{\partial^2 u_1}{\partial t^2},$$

$$(11) \quad l_3 \frac{\partial^2 u_2}{\partial x^2} + l_2 \frac{\partial^2 u_1}{\partial x \partial y} + l_1 \frac{\partial^2 u_2}{\partial y^2} - \left(1 + t_1 \delta_{2k} \frac{\partial}{\partial t}\right) \frac{\partial T}{\partial y} + \frac{\partial \varphi}{\partial y} = \frac{\partial^2 u_2}{\partial t^2},$$

$$(12) \quad \left(\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2}\right) + l_4 \left(1 + t_1 \delta_{2k} \frac{\partial}{\partial t}\right) T - l_5 \varphi - l_6 \left(\frac{\partial u_1}{\partial x} + \frac{\partial u_2}{\partial y}\right) = l_7 \frac{\partial^2 \varphi}{\partial t^2},$$

$$(13) \quad \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) - l_8 \left(1 + t_0 \delta_{1k} \frac{\partial}{\partial t}\right) \frac{\partial T}{\partial t} - l_9 \left(\frac{\partial}{\partial t} + t_0 \delta_{1k} \frac{\partial^2}{\partial t^2}\right) \left(\frac{\partial u_1}{\partial x} + \frac{\partial u_2}{\partial y}\right) - l_{10} \frac{\partial \varphi}{\partial t} + l_{11} \left(1 + t_0 \delta_{1k} \frac{\partial}{\partial t}\right) Q = 0.$$

The dimensionless constitutive relations are:

$$(14) \quad \sigma_{xx} = l_1 \frac{\partial u_1}{\partial x} + l_{12} \frac{\partial u_2}{\partial y} - \left(1 + t_1 \delta_{2k} \frac{\partial}{\partial t}\right) T + \varphi,$$

$$(15) \quad \sigma_{xy} = l_3 \left(\frac{\partial u_1}{\partial y} + \frac{\partial u_2}{\partial x}\right),$$

$$(16) \quad \sigma_{yy} = l_{12} \frac{\partial u_1}{\partial x} + l_1 \frac{\partial u_2}{\partial y} - \left(1 + t_1 \delta_{2k} \frac{\partial}{\partial t}\right) T + \varphi,$$

where, the coefficients  $l_i$  are given in Appendix-1.

### 3. Normal mode analysis

The solution of the considered physical variables can be decomposed in terms of normal mode as:

$$(u_i, u_i^e, T, \varphi, \sigma_{ij}, \sigma_{ij}^e, Q)(x, y, t) = (u_i^*, u_i^{e*}, T^*, \varphi^*, \sigma_{ij}^*, \sigma_{ij}^{e*}, Q^*)(x) e^{\omega t + i b y},$$

where,  $u_i^*(x), u_i^{e*}(x), T^*(x), \varphi^*(x), \sigma_{ij}^*(x), \sigma_{ij}^{e*}(x), Q^*$  are the amplitudes of field quantities.

Using normal mode in equation (10)–(13), we get:

$$(17) \quad (l_1 D^2 - B_1)u_1^* + ibl_2 Du_2^* - B_2 DT^* + D\varphi^* = 0,$$

$$(18) \quad ibl_2 Du_1^* + (l_3 D^2 - B_3)u_2^* - ibB_2 T^* + ib\varphi^* = 0,$$

$$(19) \quad -l_6 Du_1^* - ibl_6 u_2^* + B_2 l_4 T^* + (D^2 - B_4)\varphi^* = 0,$$

$$(20) \quad -l_9 B_6 Du_1^* - ibl_9 B_6 u_2^* + (D^2 - B_7)T^* - l_{10}\omega\varphi^* = -l_{11}B_5 Q^*,$$

$$(21) \quad \sigma_{xx}^* = l_1 Du_1^* + ibl_{12}u_2^* - B_2 T^* + \varphi^*,$$

$$(22) \quad \sigma_{yy}^* = l_{12} Du_1^* + ibl_1 u_2^* - B_2 T^* + \varphi^*,$$

$$(23) \quad \sigma_{xy}^* = l_3 (ibu_1^* + Du_2^*),$$

where, the coefficients  $B_j$  are given in Appendix 2.

Eliminating  $u_2^*(x)$ ,  $T^*(x)$ ,  $\varphi^*(x)$  from equations (17)–(20), we get:

$$(24) \quad (D^8 + AD^6 + BD^4 + CD^2 + E)u_1^*(x) = RQ^*,$$

where, the coefficients  $A, B, C, E$  and  $R$  are given in Appendix 3.

Similarly,  $u_2^*(x)$ ,  $T^*(x)$ ,  $\varphi^*(x)$  satisfies the equation:

$$(25) \quad (D^8 + AD^6 + BD^4 + CD^2 + E)(u_2^*(x), T^*(x), \varphi^*(x)) = RQ^*,$$

which can be written as:

$$(26) \quad (D^2 - k_1^2)(D^2 - k_2^2)(D^2 - k_3^2)(D^2 - k_4^2)u_1^*(x) = RQ^*,$$

where  $k_n^2$ , ( $n = 1, 2, 3, 4$ ) are the roots of characteristic equation (25).

The series solution for the physical quantities are given by:

$$(27) \quad u_1^*(x) = \sum_{n=1}^4 [M_n(b, \omega)e^{-k_n x}] + S,$$

$$(28) \quad u_2^*(x) = \sum_{n=1}^4 [M'_n(b, \omega)e^{-k_n x}] - S_1,$$

$$(29) \quad T^*(x) = \sum_{n=1}^4 [M_n''(b, \omega) e^{-k_n x}] - S_2,$$

$$(30) \quad \varphi^*(x) = \sum_{n=1}^4 [M_n'''(b, \omega) e^{-k_n x}] - S_3,$$

where,  $M_n(b, \omega)$ ,  $M_n'(b, \omega)$ ,  $M_n''(b, \omega)$ ,  $M_n'''(b, \omega)$  are specific functions, depending upon  $b$  and  $\omega$ .

Using equation (27)–(30) in equation (17)–(20), we get:

$$(31) \quad M_n'(b, \omega) = H_{1n} M_n(b, \omega),$$

$$(32) \quad M_n''(b, \omega) = H_{2n} M_n(b, \omega),$$

$$(33) \quad M_n'''(b, \omega) = H_{3n} M_n(b, \omega).$$

Using (31)–(33), the series solution takes the form:

$$(34) \quad u_2^*(x) = \sum_{n=1}^4 [H_{1n} M_n(b, \omega) e^{-k_n x}] - S_1,$$

$$(35) \quad T^*(x) = \sum_{n=1}^4 [H_{2n} M_n(b, \omega) e^{-k_n x}] - S_2,$$

$$(36) \quad \varphi^*(x) = \sum_{n=1}^4 [H_{3n} M_n(b, \omega) e^{-k_n x}] - S_3,$$

$$(37) \quad \sigma_{xx}^*(x) = \sum_{n=1}^4 [H_{4n} M_n(b, \omega) e^{-k_n x}] + S_4,$$

$$(38) \quad \sigma_{xy}^*(x) = \sum_{n=1}^4 [H_{5n} M_n(b, \omega) e^{-k_n x}] + S_5,$$

$$(39) \quad \sigma_{yy}^*(x) = \sum_{n=1}^4 [H_{6n} M_n(b, \omega) e^{-k_n x}] + S_6,$$

where, coefficients  $H_{ij}$ ,  $S$  and  $S_j$  are given in Appendix 4.

Similarly, for the elastic half space, the solutions are of the form:

$$(40) \quad u_1^{e*}(x) = \sum_{n=1}^2 [R_n(b, \omega) e^{r_n x}],$$

$$(41) \quad u_2^{e*}(x) = \sum_{n=1}^2 [R'_n(b, \omega) e^{r_n x}],$$

where  $R_n(b, \omega)$  and  $R'_n(b, \omega)$  are specific functions, depending upon  $b$  and  $\omega$  and  $r_n^2$ , ( $n = 1, 2$ ) are roots of the equation:

$$(42) \quad (D^4 - GD^2 + L)u_1^{e*}(x) = 0.$$

And solutions of physical quantities are given by:

$$(43) \quad u_2^{e*}(x) = \sum_{n=1}^2 [L_{1n} R_n(b, \omega) e^{r_n x}],$$

$$(44) \quad \sigma_{xx}^{e*}(x) = \sum_{n=1}^2 [L_{2n} R_n(b, \omega) e^{r_n x}],$$

$$(45) \quad \sigma_{yy}^{e*}(x) = \sum_{n=1}^2 [L_{3n} R_n(b, \omega) e^{r_n x}],$$

$$(46) \quad \sigma_{xy}^{e*}(x) = \sum_{n=1}^2 [L_{4n} R_n(b, \omega) e^{r_n x}],$$

where, the coefficients  $G$ ,  $L$  and  $L_{kj}$  are given in Appendix 5.



#### 4. Applications

We suppress the positive exponentials in the physical problem to determine the parameters  $M_n$ , ( $n = 1, 2, 3, 4$ ) and  $R_n$ , ( $n = 1, 2$ ), which are unbounded at infinity. Constants  $M_1, M_2, M_3, M_4$  and  $R_1, R_2$  have to be selected, such that boundary conditions at the surface  $x = 0$  are:

$$\sigma_{xx} = \sigma_{xx}^e - P_1 e^{\omega t + i b y}, \quad u_1 = u_1^e, \quad u_2 = u_2^e, \quad \sigma_{xy} = \sigma_{xy}^e, \quad \varphi = 0, \quad \frac{\partial T}{\partial x} = 0.$$

where  $P_1$  is the magnitude of the mechanical force.

Using the expressions of  $\sigma_{xx}$ ,  $\sigma_{xx}^e$ ,  $u_1, u_1^e, u_2, u_2^e, \sigma_{xy}, \sigma_{xy}^e, T, \varphi$  into above boundary conditions, we get:

$$\begin{aligned} \sum_{n=1}^4 [H_{4n} M_n] - \sum_{n=1}^2 [L_{2n} R_n] &= -P_1 - S_4, & \sum_{n=1}^4 [M_n] - \sum_{n=1}^2 [R_n] &= -S, \\ \sum_{n=1}^4 [H_{1n} M_n] - \sum_{n=1}^2 [L_{1n} R_n] &= -S_1, & \sum_{n=1}^4 [H_{5n} M_n] - \sum_{n=1}^2 [L_{4n} R_n] &= -S_5, \\ \sum_{n=1}^4 [H_{3n} M_n] &= S_3, & \sum_{n=1}^4 [H_{2n} k_n M_n] &= 0. \end{aligned}$$

After solving the above system of non homogeneous equations, we get the values of constant  $M_1, M_2, M_3, M_4, R_1, R_2$  and hence, obtain the components of normal displacement, normal force stress, temperature distribution and microelongation at the interface of microelongated thermoelastic half space and elastic half space.

#### Special Cases

1. If we neglect microelongation effect i. e.  $\lambda_0 = \beta_1 = \lambda_1 = a_0 = j_0 = 0$ , we obtain the results for thermoelastic solid (TS).

2. Letting  $Q \rightarrow 0$  in (7) and  $\mu^e \rightarrow 0$  in (8) and (9), problem reduces to a plane strain problem in thermoelastic microelongated solid with an overlying infinite non viscous fluid [34].

#### 5. Numerical results and discussions

For numerical computations, we consider the values of constants for aluminium epoxy-like material as [28]:

$$\lambda = 7.59 \times 10^{10} \text{ N/m}^2, \quad \mu = 1.89 \times 10^{10} \text{ N/m}^2, \quad a_0 = 0.61 \times 10^{-10} \text{ N},$$

$$\rho = 2.19 \times 10^3 \text{ kg/m}^3, \quad \beta_1 = 0.05 \times 10^5 \text{ N/m}^2\text{K}, \quad \beta_0 = 0.05 \times 10^5 \text{ N/m}^2\text{K},$$

$$C^* = 966 \text{ J/(kgK)}, \quad K^* = 252 \text{ J/msK}, \quad j_0 = 0.196 \times 10^{-4} \text{ m}^2,$$

$$\lambda_0 = \lambda_1 = 0.37 \times 10^{10} \text{ N/m}^2, \quad t_0 = 0.01, \quad t_1 = 0.0001, \quad T_0 = 293 \text{ K}.$$

The physical constants for elastic medium (granite) as [33]:

$$\lambda^e = 0.884 \times 10^{10} \text{ N/m}^2, \quad \mu^e = 1.2667 \times 10^{10} \text{ N/m}^2, \quad \rho^e = 2.6 \times 10^3 \text{ Kg/m}^3.$$

The computations are carried out for the value of non-dimensional time  $t = 0.2$  in the range of  $0 \leq y \leq 10$  and on the surface  $x = 1.0$ . The numerical values for normal displacement, normal force stress, temperature distribution and microelongation are shown in Figs 2–5 for Green Lindsay (GL) theory  $\delta_{1k} = 0, \delta_{2k} = 1$  and mechanical force with magnitude:  $P_1 = 1.0, \omega = \omega_0 + \iota\xi, \omega_0 = -0.2, \xi = 0.1$  and  $b = 0.8$ .

(a) Thermoelastic microelongated solid (TMS) with  $Q = 1.0$  by solid line with dashed symbol  $\blacklozenge$ .

(b) Thermoelastic microelongated solid (TMS) with  $Q = 10.0$  by dashed line with centered symbol  $\blacksquare$ .

(c) Thermoelastic solid (TS) with  $Q = 1.0$  by dashed line with centered symbol  $\blacktriangle$ .

(d) Thermoelastic solid (TS) with  $Q = 10.0$  by dotted line with centered symbol  $\times$ .

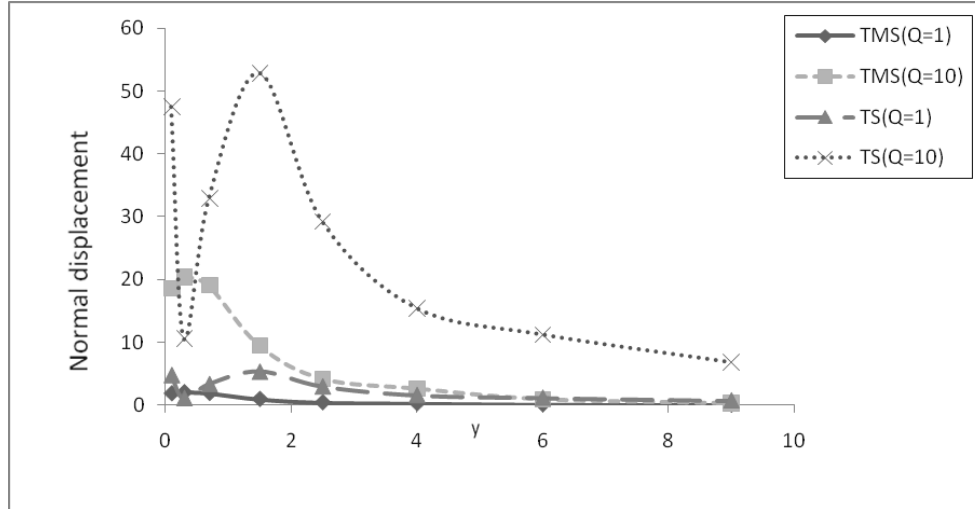


Fig. 2. Normal displacement variation with horizontal distance

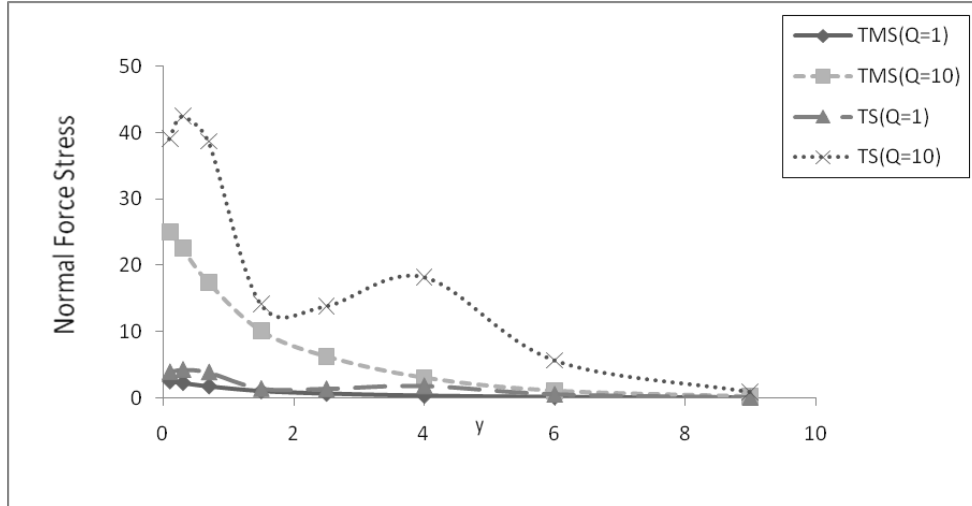


Fig. 3. Normal force stress variation with horizontal distance

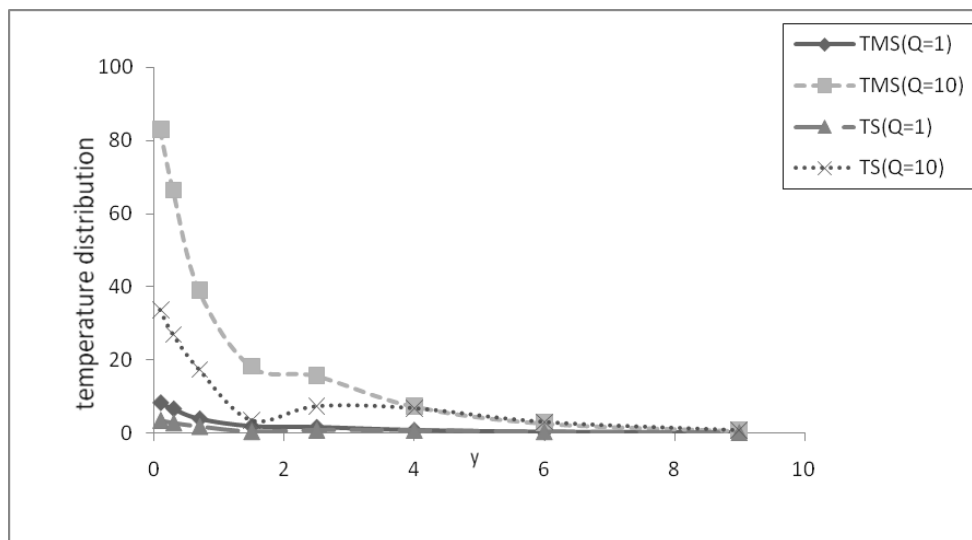


Fig. 4. Temperature distribution variation with horizontal distance

## 6. Discussion

As expected, the values of normal displacement, normal force stress, temperature distribution, and microelongation near the point of application of source are bigger, when the magnitude of internal heat source is larger

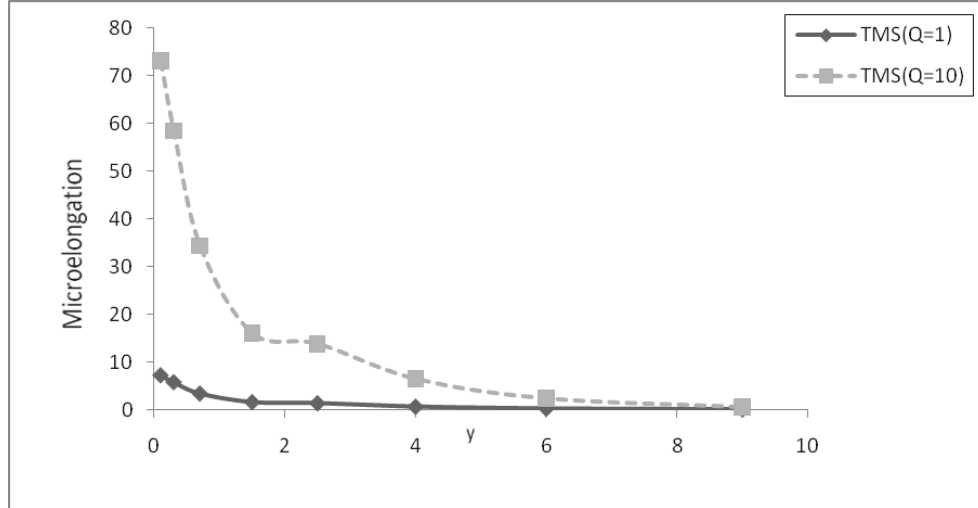


Fig. 5. Microelongation variation with horizontal distance

( $Q = 10.0$ ) as compared to that, when the magnitude of internal heat source is ( $Q = 1.0$ ) in the range of  $0 \leq y \leq 10.0$ . From the Figs 2 and 3, it is very much clear that normal displacement and normal force stress vary in similar manner. The variation of all the quantities tends to zero as horizontal distance increases. These variations of normal displacement, temperature distribution, normal force stress and microelongation are shown in Figs 2–5, respectively.

## 7. Conclusion

1. Microelongation and internal heat source have appreciable effect on all quantities.
2. The magnitude of quantities in the neighbourhood of mechanical force increases with increase in magnitude of internal heat source.
3. The problem may also be discussed in context of mechanical force (i.e.  $Q \rightarrow 0$ ) and internal heat source (i.e.  $P_1 \rightarrow 0$ ), separately. The graphical results of these expressions may also be discussed in similar way.
4. The problem finds wide applications in dynamics and thermoelastic theory, including solid-liquid crystals, composite reinforced materials and porous media.

List of used symbols and notations	List of used abbreviations
$\sigma = \sigma_{kk}$	microelongational stress tensor
$s = s_{kk}$	component of stress tensor
$\alpha_{t_1}$ and $\alpha_{t_2}$	coefficient of linear thermal expansion,
$a_0, \lambda_0, \lambda_1$	microelongational constants
$t_0, t_1$	thermal relaxation times
$\rho$	density of microelongated medium
$j_0$	Microinertia
$K^*$	coefficient of thermal conductivity
$C^*$	specific heat at constant strain
$m_k$	component of microstretch vector
$T$	thermodynamic temperature above reference temperature $T_0$
$\varphi$	microelongational scalar
$\lambda, \mu$	lame's constants
$\delta_{kl}$	kronecker delta
$\vec{u}$	displacement vector of microelongated solid
$Q$	internal heat source
$\vec{u}^e$	displacement vector of elastic solid
$\lambda^e, \mu^e$	elastic constants
$\rho^e$	density of elastic solid
$\omega$	complex frequency
$b$	wave number in $y$ -direction

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## Appendix 1

$$l_1 = \frac{(\lambda + 2\mu)}{\rho c_1^2},$$

$$l_2 = \frac{(\lambda + \mu)}{\rho c_1^2},$$

$$l_3 = \frac{\mu}{\rho c_1^2},$$

$$l_4 = \frac{\beta_1 \lambda_0 c_1^2}{a_0 \omega^* \beta_0},$$

$$l_5 = \frac{\lambda_1 c_1^2}{a_0 \omega^*},$$

$$l_6 = \frac{\lambda_0^2}{\rho a_0 \omega^*},$$

$$l_7 = \frac{\rho j_0 \omega^* c_1^2}{2a_0},$$

$$l_8 = \frac{\rho C^* c_1^2}{K^* \omega^*},$$

$$l_9 = \frac{\beta_0^2 T_0}{K^* \omega^* \rho},$$

$$l_{10} = \frac{\beta_0 \beta_1 T_0 c_1^2}{K^* \omega^* \lambda_0},$$

$$l_{11} = \frac{\rho c_1^4}{K^* \omega^* T_0},$$

$$l_{12} = \frac{\lambda}{\rho c_1^2}.$$

## Appendix 2

$$D \equiv \frac{d}{dx},$$

$$B_1 = \omega^2 + l_3 b^2,$$

$$B_2 = (1 + t_1 \delta_{2k} \omega),$$

$$B_3 = \omega^2 + l_1 b^2,$$

$$B_4 = b^2 + l_5 + l_7 \omega^2,$$

$$B_5 = (1 + t_0 \delta_{1k} \omega),$$

$$B_6 = \omega(1 + t_0 \delta_{1k} \omega),$$

$$B_7 = b^2 + l_8 B_5 \omega.$$



**Appendix 3**

$$\begin{aligned}
A &= \frac{-1}{l_1 l_3} [l_1 l_3 (B_4 + B_7) - l_1 B_3 + l_3 B_1 + l_3 l_6 + B_2 l_3 l_9 B_6 + b^2 l_2^2] \\
B &= \frac{-1}{l_1 l_3} [-l_1 B_2 l_4 l_{10} l_3 \omega + l_1 l_3 B_4 B_7 + l_1 B_3 (B_4 + B_7) - l_1 b^2 B_2 B_6 l_9 + b^2 l_1 B_6 \\
&\quad - B_1 l_3 (B_4 + B_7) + B_1 B_3 - b^2 l_2^2 (B_4 + B_7) + l_3 l_6 l_{10} B_2 \omega - l_3 l_9 B_2 B_4 B_6 \\
&\quad - l_9 B_2 B_3 B_6 - l_3 l_6 B_7 - l_3 l_4 l_9 B_2 B_6 - B_3 l_6] \\
C &= \frac{-1}{l_1 l_3} [B_2 B_3 l_1 l_4 l_{10} \omega + B_3 B_4 B_7 l_1 - b^2 l_1 l_6 l_{10} B_2 \omega + b^2 l_1 l_9 B_2 B_4 B_6 - b^2 l_1 l_6 B_7 \\
&\quad - b^2 l_1 l_4 l_9 B_2 B_6 + B_1 B_2 l_3 l_4 l_{10} \omega^2 - l_3 B_1 B_4 B_7 + B_1 B_3 (B_4 + B_7) + b^2 B_1 B_2 B_6 l_9 \\
&\quad + b^2 l_2^2 B_2 l_4 l_{10} \omega + b^2 l_2^2 B_4 B_7 - 2b^2 B_7 l_2 l_6 - 2b^2 B_2 B_6 l_2 l_4 l_9 - l_6 l_{10} B_2 B_3 \omega \\
&\quad + B_2 B_3 B_4 B_6 l_9 + B_3 B_7 l_6 + B_2 B_3 B_6 l_4 l_9 - b^2 l_6 B_1] \\
E &= \frac{-1}{l_1 l_3} [-l_4 l_{10} B_1 B_2 B_3 \omega - B_1 B_3 B_4 B_7 + b^2 l_6 l_{10} B_1 B_2 \omega - b^2 l_9 B_1 B_2 B_4 B_6 \\
&\quad + b^2 l_6 B_1 B_7 + b^2 l_4 l_9 B_1 B_2 B_6] \\
R &= b^2 l_{11} B_1 B_2 B_4 B_5 (l_4 - B_4)
\end{aligned}$$

**Appendix 4**

$$\begin{aligned}
H_{1n} &= \frac{ib[(l_1 - l_2)k_n^2 - B_1]}{[(B_3 - b^2 l_2)k_n - l_3 k_n^3]}, \\
H_{2n} &= \frac{[l_3 k_n^4 - (B_4 l_3 + B_3)k_n^2 + (B_3 B_4 - b^2 l_6)]H_{1n} - ib[l_2 k_n^3 - (l_2 B_4 - l_6)k_n]}{ib[B_2(k_n^2 - B_4) + B_2 l_4]}, \\
H_{3n} &= \frac{(l_1 k_n^2 - B_1 - ibl_2 k_n H_{1n} + B_2 k_n H_{2n})}{k_n}, \\
H_{4n} &= ibl_{12} H_{1n} - B_2 H_{2n} + H_{3n} - l_1 k_n, \\
H_{5n} &= l_3 (ib - k_n H_{1n}), \\
H_{6n} &= ibl_1 H_{1n} - B_2 H_{2n} + H_{3n} - l_{12} k_n, \\
S &= \frac{RQ^*}{E},
\end{aligned}$$

$$S_1 = ibB_1S,$$

$$S_2 = ibB_1(B_3B_4 - b^2l_6)S,$$

$$S_3 = B_1S,$$

$$S_4 = -(B_2S_2 - S_3 + ibl_{12}S_1),$$

$$S_5 = ibl_3S,$$

$$S_6 = -(B_2S_2 - S_3 + ibl_1S_1).$$

### Appendix 5

$$G = \frac{b^2(a_1^2 + a_3^2 - a_2^2) + (a_1 + a_3)\omega^2}{a_1a_3},$$

$$L = \frac{b^4a_1a_3 + b^2\omega^2(a_1 + a_3) + \omega^4}{a_1a_3},$$

$$a_1 = \frac{\lambda^e + 2\mu^e}{\rho^e c_1^2},$$

$$a_2 = \frac{\lambda^e + \mu^e}{\rho^e c_1^2},$$

$$a_3 = \frac{\mu^e}{\rho^e c_1^2},$$

$$L_{1n} = \frac{\omega^2 + b^2a_3 - a_1r_n^2}{iba_2r_n},$$

$$L_{2n} = \frac{(\lambda^e + 2\mu^e)r_n + ib\lambda^e L_{1n}}{\rho c_1^2},$$

$$L_{3n} = \frac{\lambda^e r_n + ib(\lambda^e + 2\mu^e)L_{1n}}{\rho c_1^2},$$

$$L_{4n} = \frac{\mu^e(ib + r_n L_{1n})}{\rho c_1^2}.$$