



DOI 10.1515/msd-2015-0006

ACTIVATED SLUDGE SYSTEMS AS DIFFERENTIAL EQUATIONS: A FRESH VIEW ON THE CLASSICAL MODELS FOR WASTEWATER TREATMENT PLANTS

Horea, OLOSUTEAN¹ and Letiția, OPREAN²¹„Lucian Blaga” University of Sibiu, Sibiu, Romania, mesaje.facultate@yahoo.com²„Lucian Blaga” University of Sibiu, Sibiu, Romania, letitia.oprean@ulbsibiu.ro

ABSTRACT: Present in the theory and practice of wastewater treatment for over 50 years in consistent numbers, activated sludge models are widely used by plant operators. The early stage models, based on differential equations were synthesized and reinterpreted, presenting new insides that can help to a better understanding of the processes involved in the wastewater treatment. A consistent symbology is presented, for a much homogenous perspective, and the obvious similarities between several equations from the models are pointed out. The incomplete information from the models is emphasized, showing their amount of providing supplementary data for plant operators and researchers in the field, as well as the causes leading to their drawbacks and the effects derived from these drawbacks.

1. INTRODUCTION

Ross E. McKinney's 1962 model marked the start of activated sludge modeling, with over 15 different mathematical approaches describing the biological interaction taking place inside the activated sludge systems (Ognean and Vaicum, 1987, Henze et al., 2000).

The early attempts, generated in the 60's and 70's, were made by individual researchers (mostly engineers), and the results are constructed as differential equations based on Monod's concepts, mostly describing the biomass fluxes of the system, the so called "conceptual equations" (Olosutean and Oprean, 2011): the organic matter balance (substrate accumulation = material entering the system – material removed from the system – material consumed in reaction) and the bacterial – or, more correct, the activated sludge – balance (bacterial mass accumulation = synthesized bacterial mass – endogenous consumption of bacterial mass – bacterial mass removed from the system). The mathematical definition of the terms from the conceptual equations is specific to each researcher, and some of the models were developed fractionally over several years, the model never being published as a whole, therefore some inconsistencies are present in most models.

Prior to the formation of the Task Group on Mathematical Modeling for Design and Operation of Activated Sludge Processes by the IAWRC, and the creation of "state-of-art" models (Henze et al., 1986; 1987, 1999, 2000; Gujer et al., 1995, 1999; Jeppsson, 1996), the conceptual equations generated by McKinney (1962), Lawrence and McCarty (1970), Eckenfelder (1971), Goodman and Englade (1974), Grau et al. (1975), Gaudy (Ramanathan and Gaudy, 1971; Gaudy and Srinivasaraghavan, 1974; Srinivasaraghavan and Gaudy, 1975; Gaudy and Kincannon, 1977)), Christoulas and Tebbut (1976) or Jones

(1978) were the mathematical bases for wastewater plant operators, despite their obvious drawbacks.

2. THE MODELS

Most of the named researchers came up with two conceptual equations for the organic matter balance (equations [1], for the McKinney Model, [2], for the Eckenfelder Model, [3], for the Gaudy Model, and [4], for the Grau Model) and for the activated sludge balance (equations [5] and [6], for the McKinney Model, [7], for the Eckenfelder Model, [8], for the Lawrence and McCarty Model, [9], for the Gaudy Model, and [10] and [11], for the Grau Model), the symbology being presented in Table 1.

Table 1. Symbols used in the equations.

Notations	
C	concentration of substances from the aeration basin
C0	concentration of substances from the influent (initial concentration)
Q	influent capacity
M	mass of active bacteria
E	endogenous metabolism products
N	total mass of suspensions from the aeration basin
X	concentration of active bacteria
T	hydraulic retention time
V	volume of the aeration basin
Y	bacterial growth yield
b	endogenous consumption constant
t	time
x	recirculation fraction
w	influent fraction removed with excess sludge
s	sedimentation coefficient expressing the concentration of suspensions removed with excess sludge

k/K	process constants
μ_{max}	specific biomass growth speed

$$V * \frac{dC}{dt} = Q * C_0 - Q * C - k * V * C \quad [1]$$

$$V * \frac{dC}{dt} = Q * C_0 - Q * C - v * V \quad [2]$$

$$V * \frac{dC}{dt} = Q * C_0 - Q * C - \frac{\mu_{max} * X * C * V}{Y * (K_s + C)} \quad [3]$$

$$V * \frac{dC}{dt} = C_0 * Q - k * X * \frac{C}{C_0} - C * Q \quad [4]$$

$$V * \frac{dM}{dt} = k * V * C - b * V * M - Q * M \quad [5]$$

$$V * \frac{dM}{dt} = k * V * C - b * V * M - Q * x * M \quad [6]$$

$$V * \frac{dM}{dt} = v * Y * V - b * M * V - Q * M \quad [7]$$

$$V * \frac{dX}{dt} = (Y * \frac{dC}{dt} - b * X) * V - (Q_w * X_r + Q * X_e - Q_w * X_e) \quad [8]$$

$$V * \frac{dX}{dt} = q * X_r - b * X * V + \frac{\mu_{max} * X * C * V}{Y * (K_s + C)} - (Q + q) * X \quad [9]$$

$$V * \frac{dX}{dt} = Y * k * X * V * \frac{C_0}{C} - b * V * X - Q * X \quad [10]$$

$$V * \frac{dX}{dt} = (Y * k * X * \frac{C}{C_0} - b * X) * V + q * X_r - (Q + q) * X$$

The conceptual equations were used for the extraction of the most important parameters of the wastewater system, mainly C , M , N and E , by reducing the equations to the stationary state, in which the left term equals 0.

3. DISCUSSIONS

3.1. The problem of research independence

The first obvious problem of the early wastewater treatment models is related to the relative independence of the research made by the scientist involved in the field in the 60's and 70's. Each of the researchers was at least partially unaware of the other's work, since the information was travelling at a much lower speed than in the recent days, and in a much different form than it is now. Therefore, there is a strong incoherence in the way the models were constructed, especially in the way each researcher choose the notations for the system's variables, leading to a higher amount of variation and to an apparent individuality and originality of the models. For example, McKinney used the notation M_a for the active bacterial biomass, while Eckenfelder used M_b for the same variable, using M_a for the active volatile mass, making way for confusion among plant operators and researchers working in the field. We used the similar notations for equations [1] to [11], making them comparable for further analysis.

3.2. Originality and similarity

At first sight, each of the models is encompassing the researcher's own vision regarding the activated sludge system, but a close analysis shows that most of the elements from the equations are having a higher degree of similarity, and the originality is apparent, derived mostly by the way each researcher theoretically described the system and the use of notations.

If we look at the equation describing the organic matter balance (Table 2), we can see that the first two terms are identical, although the equations are derived starting from a different point of view by each scientist or research group (Ognean and Vaicum, 1987), the only difference being found in the term describing the material consumed in reaction.

Table 2. Organic matter balance equations in classical models.

Model	material entering the system	material removed from the system	material consumed in reaction
<i>McKinney</i>	$Q * C_0$	$Q * C$	$k * V * C$
<i>Eckenfelder</i>	$Q * C_0$	$Q * C$	$v * V$
<i>Gaudy</i>	$Q * C_0$	$Q * C$	$\frac{\mu_{max} * X * C * V}{Y * (K_s + C)}$
<i>Grau et al.</i>	$Q * C_0$	$Q * C$	$k * X * \frac{C}{C_0}$

Although they seem rather different, there are just differently constructed, and the rearrangement of the terms is showing that the ideas of the researchers are not that different.

For example, if we transform the term from Eckenfelder's equation by dividing and multiplying to C , we obtain:

$\frac{v}{C} * V * C$, or $\frac{v}{C} * V * C$, the difference to McKinney's term being that k , a process constant, is replaced by v/C , which is also a constant, since the growth speed of the bacterial mass is dependent on the amount of organic substance in the bioreactor. The same principle applies on Gaudy's, with $\frac{\mu_{max} * X}{Y * (K_s + C)}$ also being

constant, because of the dependence between μ_{max} and Y , and between X and C , respectively.

The same situation is applicable for the equations derived from the conceptual equations that are defining M and C . Such equations were presented by McKinney (equations [12] and [15]), Eckenfelder (equations [13] and [16]), and Goodman and Englande (equations [14] and [17]), with X_v being the concentration of volatile substances from the activated sludge.

$$C = \frac{C_0}{k * T + 1} \quad [12]$$

$$C = \frac{C_0}{k * X_v * T + 1} \quad [13]$$

$$C = \frac{C_0}{k * T + 1} \quad [14]$$

$$M = \frac{k * C}{\frac{1}{T} + b} \quad [15]$$

$$M = \frac{Y * k * X_v * C}{\frac{1}{T} + b} \quad [16]$$

$$M = \frac{Y * k * C}{\frac{1}{T} + b} \quad [17]$$

Again, if we look at the equations given for the concentration of substances from the aeration basin, it is obvious that all three models have the same formula, the difference being on the denominator, where the term k , a constant from the McKinney and Goodman and Englande models, is replaced by $k * X_v$ by Eckenfelder. Since $k * X_v$ is also constant, due to the relative constancy of volatile substances in the aeration tank, the formulas are highly similar.

The equations for the mass of active bacteria are again, highly similar, the difference being that the Eckenfelder Model and the Goodman and Englande Model are containing the term Y , bacterial growth yield, as opposed to the McKinney Model. However, since the constant present in equation [15] is described by McKinney as „bacterial synthesis constant” it is presumable that this constant is dependent on the bacterial growth yield, making all the formulas practically the same.

3.3. How many models there are?

The similarities between the models raises the question if each of the models is a separate entity, or they are just different facades of the same mathematical problematic. Since the conceptual equations are highly similar and the formulas for systems parameters derived from the equations are practically the same, it would seem that the presented models are just different approaches of the same idea and they should be considered as a single mathematical entity. The Goodman and Englande Model, presented as an individual construction, although the authors only unified and standardized the information from the McKinney and Eckenfelder models, with

the agreement of both McKinney and Eckenfelder (Ognean and Vaicum, 1987) is a further proof in that direction.

However, the main element of originality is to be found in the basis of each model. Although mathematically similar, they are the result of completely different concepts about the wastewater system, concepts that were all proven correct, since the mathematical results were similar. Each new concept added little more to the information previously known, making the wastewater system easier and easier to understand by plant operators and researcher. The information found in these concepts was useful in the construction of more advanced models in a more recent time period, models that are widely used in the recent day’s wastewater management (Vanrolleghem et al., 2003).

Therefore, the originality of the models is dependent on the point of view we use in analyzing them. If reduced to basic concepts, they are individual research ideas where significantly different visions are applied to the same structure. If reduced to the conceptual equations and to the formulas obtained for several system parameters, they are rather similar, although the way the equations and parameters were obtained are distinct.

3.4. The hidden information

Being mostly the result of several years of tryouts from a researcher or even a group of researchers, the final result, or the model itself, is usually made of fragments from different papers, some parts of the models being not described in detail, and some terms of different equations, although referring to the same element, being constructed differently (Oprean and Olosutean, 2011; Olosutean and Oprean, 2011).

One of the main problems of the mathematical derivation of most of the models is the fact that they are usually incomplete. Each researcher provided conceptual equations, but some limited to the information regarding the concentration of substances from the aeration tank (Christoulas and Tebbut, 1976; Jones, 1978), while others focused also on the active bacteria from the system, providing equations for the balances of either the mass, or the concentration of active bacteria, and never for both the variables. From that point of view, all the models are to be considered incomplete, since the amount of information still derivable from the initial concept is significant.

Even more, the formulas that are given by some of the models are not completely derived. Gaudy’s formula for the concentration of organic substances from the aeration tank is quite complicated and can be significantly reduced:

$$C = \frac{K_s * (\frac{V}{T * T_c * Q} + b)}{\mu_{max} - \frac{V}{T * T_c * Q} + b} \quad [18]$$

Continuing the derivation by reducing to a common denominator, we have:

$$C = \frac{K_s * V + K_s * b * T * T_c * Q}{(\mu_{max} + b) * T * T_c * Q - V} \quad [19]$$

T being V/Q , by replacing and reducing Q , we have:

$$C = \frac{K_s * V + K_s * b * V * T_c}{(\mu_{max} + b) * V * T_c - V} \quad [20]$$

If we reduce V in equation [20], we obtain a much simpler equation, depending on only four elements: K_s , T_c , μ_{max} și b :

$$C = \frac{K_s * (1 + b * T_c)}{(\mu_{max} + b) * T_c - 1} \quad [21]$$

Gaudy's formula for volume calculation can be reduced significantly with the use of simple mathematical operations (Olosutean and Oprean, 2011):

$$V = \frac{Y * Q * [C_0 - (1 + r) * C] + r * X_r * Q}{b * X} - \frac{(1 + r) * Q}{b} \quad [22]$$

$$V = \frac{Y * Q * [C_0 - (1 + r) * C] + r * X_r * Q - (1 + r) * X * Q}{b * X} \quad [23]$$

$$V = \frac{Q * [Y * C_0 - (1 + r) * Y * C + r * X_r - (1 + r) * X]}{b * X} \quad [24]$$

$$V = \frac{Q * [Y * C_0 + r * X_r - (1 + r) * (Y * C + X)]}{b * X} \quad [25]$$

Jones's model is also incompletely derived. It provides a conceptual equation for the organic matter balance, but does not derive the formula for the concentration of substances from the aeration basin. Applying the stationary condition, we obtain:

$$\frac{\mu_{max} * X_v * C}{Y * K_s + C} = -V * \frac{C}{K_M + C} * X_a \quad [22]$$

If we simplify by C and rearrange the terms of the equation, we have:

$$\mu_{max} * X_v * K_M - V * X_a * Y * K_s = C * \mu_{max} * X_v - C * V * X_a * Y \quad [23]$$

The formula for the concentration of organic substances in the aeration tank is than easily extractable:

$$C = \frac{\mu_{max} * X_v * K_M - V * X_a * Y * K_s}{\mu_{max} * X_v - V * X_a * Y} \quad [24]$$

Such examples can be found in most of the models from the early period of activated sludge modeling, making the models perfectible and raising the problem of their full use in the practice of wastewater management.

3.5. The main drawbacks

Beyond the problems depicted above, the early mathematical models of the wastewater treatment are having a few consistent drawbacks, which made them insufficient and urged the creation of "state-of-art" models or ASMs.

The most important of these drawbacks is the development of the models without taking into concern the concept of „wastewater characterisation” (Sollfrank and Gujer, 1991), developed after the models were always generated. Considering the entire organic matter from the system as homogenous, and, therefore, uniform in reacting inside the aeration tank, the models are having experiencing large amounts of variation depending on the provenience of the wastewater, and the confidence limits of the variables involved were very high. The new models created by the IAWRC and its Task Group started with the separation of the active and inactive fractions from the

wastewater, being, from that reason, much more accurate and easier to be applied in practice.

The early models were considering the systems as unitary also from the point of climatic conditions, with temperature and pH not taken into concern as variables of the system. Later studies determined that these two variables are of main importance in the performance of the system, making their absence from the models a major liability (Henze et al., 2000; Vanrolleghem et al., 2003).

Maybe the most fundamental failure of the early models is the construction of the treatment system based on the model, their main goal being to provide treatment plants construction parameters. Such a concept might be viable at a certain point, but the subsequent evolution of the treatment plant, with increasing quantities of wastewater and organic loads led to system malfunctions and to a lower capacity of improving facilities constructed for largely fixed conditions.

Another problem of the differential models is the perception of the aeration basin as a system without sludge recirculation and excess sludge removal, as were most of the plants in the 60's and 70's. The authors proved that such models can be constructed as particular situation of the system with sludge recirculation and excess sludge removal, making the models more complete and applicable in the recent period, where the large majority of wastewater plants are using systems that recirculate sludge and remove the excess.

As stated before, another major drawback is the fragmentation of the equations and derivations from the models in several papers by the authors, as well as the difficulty of information circulation in the period they were developed. That factor made the models to be subjected to a certain amount of error, and resulted in largely incomplete mathematical constructions (Ognean and Vaicum, 1987; Olosutean and Oprean, 2011; Oprean and Olosutean, 2011)

4. CONCLUSIONS

The mathematical models of the activated sludge process developed in the 60's and 70's by several independent researchers were the basis for design and operation of wastewater treatment plant for a consistent period of time.

However, they proved obsolete at a certain point in time, due to their limitations and drawbacks and to the increasing quantities and heterogeneity of wastewater. The models were containing sufficient information for the development of the modern modeling methods, and their capacity of providing such information is not completely consumed.

The equations that compose the models are still interpretable and derivable in order to obtain more data about the system, although much of the equations are similar for some of the models, due to the difficulty in exchanging information at the time they were developed.

Because most of the models were developed over several years, and sometimes by different research teams, the information is prone to transcription errors, and the models are rarely found complete in a single paper.

Wastewater characterization and the development of models applicable to any treatment plant, not only the ones designed with parameters extracted from the models made the use of differential models highly difficult, and transformed them into a theoretical research base for a better understanding of the way activated sludge systems work.

5. REFERENCES

1. Christoulas D.G., Tebbut, T.H.Y., Mathematical model of a complete-mix activated-sludge plant, *Water Research*, Vol. 10, pp. 797-803 (1976).
2. Eckenfelder, W.W. Jr., *Activated sludge and extended aeration, process design in water quality engineering - new concepts and developments*, Vanderbilt Univ., Nashville, Tenn. (1971).
3. Gaudy, A.F., Jr., Kincannon, D.F., Comparing design models for activated sludge, *Water Sewage Works*, Vol. 123 (1977).
4. Gaudy, A.F., Jr., Srinivasaraghavan, R., Experimental studies on kinetic model for design and operation of activated sludge processes, *Biotechnology and Bioengineering*, Vol. 16 (1974).
5. Gernaey, K. W., Loosdrecht, M. C. M. van, Henze, M., Lind, M., Jorgensen, S. B., Activated sludge wastewater treatment plant modelling and simulation: State of the art, *Environmental Modelling & Software*, Vol. 19, pp. 763–783 (2004).
6. Goodman B.L., Englande, A.J., Jr., A unified model of the activated sludge process, *Journal of Water Pollution Control Federation*, Vol. 46 (1974).
7. Grau, P., Dohányos, M., Chudoba, J., Kinetics of multicomponent substrate removal by activated sludge, *Water Research*, Vol. 9, Nr. 7, pp. 637-642 (1975).
8. Gujer, W., Henze, M., Mino, T., Matsuo, T., Wentzel, M. C., Marais, G.V.R., The activated sludge model no. 2: Biological phosphorus removal, *Water Science and Technology*, Vol. 31, Nr. 2, pp. 1–11 (1995).
9. Gujer, W., Henze, M., Mino, T., Loosdrecht, M. van, Activated sludge model no. 3, *Water Science and Technology*, Vol. 39, Nr. 1, pp. 183–193 (1999).
10. Henze, M., Gujer, W., Mino, T., Matsuo, T., Wentzel, M. C., Marais, G.V.R., Loosdrecht, M. van, Activated sludge model no. 2d, ASM2D, *Water Science and Technology*, Vol. 39, Nr. 1, pp. 165–182 (1999).
11. Henze, M., Gujer, W., Mino, T., Loosdrecht, M. van, *Activated Sludge Models ASM1, ASM2, ASM2d and ASM3*, IWA Publishing (2000).
12. Henze, M., Grady, C.P.L., Gujer, W., Marais, G.V.R., Matsuo, T., Activated sludge model no. 1., *IAWPRC Task Group on Mathematical Modelling for Design and Operation of Biological Wastewater Treatment Processes, Scientific and Technical Report 1*, IAWPRC, London (1986).
13. Henze, M., Grady, C. P. L., Gujer, W., Marais, G. V. R. and Matsuo, T., A general model for single-sludge wastewater treatment systems, *Water Research*, Vol. 21, nr. 5, pp. 505–515 (1987).
14. Jeppsson, U., *Modelling aspects of wastewater treatment processes*, Ph.D. Thesis, Lund Institute of Technology (1996).
15. Jones, G.L., A mathematical model for bacterial growth and substrate utilisation in the activated sludge process, in A. James (Ed.), *Mathematical models in water pollution control*, John Willey and Sons, New York (1978).
16. Lawrence, A.W., McCarty, P.L., Unified theory for biological treatment design and operation, *Journal of the Sanitary Engineering Division ASCE*, Vol. 96, pp. 757–78 (1970).
17. McKinney, R. E., Mathematics of completely-mixing activated sludge, *Journal of ASCE Proceeding (EE)*, Vol. 88, p. 87 (1962).
18. Ognean, T., Vaicum, L.M., *Modelarea proceselor de epurare biologică*, Ed. Acad. R.S.R., București (1987).
19. Olosutean, H., Oprean, L., Differential or matrix: the activated sludge modeling dilemma, *Management of Sustainable Development*, Vol. 3, Nr. 2, pp. 51-56 (2011).
20. Oprean, L., Olosutean, H., A new perspective on McKinney's wastewater model, *Transylvanian Review of Systematical and Ecological Research*, Vol. 12, pp. 181-192 (2011).
21. Ramanathan, M., Gaudy, A.F., Jr., Steady-state model for activated sludge with constant recycle sludge concentration, *Biotechnology and Bioengineering*, Vol. 13, pp.125-145 (1971).
22. Sollfrank, U., Gujer, W., Characterisation of domestic wastewater for mathematical modelling of the activated sludge process, *Water Science and Technology*, Vol. 27, pp. 1057–1066 (1991).
23. Srinivasaraghavan, R., Gaudy, A.F., Jr., Operational performance of an activated sludge process with constant sludge feedback, *Journal of the Water Pollution Control Federation*, Vol. 47, pp. 1946-1960 (1975).
24. Vanrolleghem P.A., Insel, G., Petersen, B., Sin, G., de Pauw, D., Nopens, I., Dovermann, H., Veijers, S., Gernaey, K.V., A comprehensive model calibration procedure for activated sludge models, *Proceedings of the Water Environment Federation*, WEFTEC 2003: Session 31 through Session 40, pp. 210-237 (2003).